

THE ~~Lib.~~
General LAWS
OF
Nature and Motion;

WITH THEIR
Application to *Mechanicks.*

ALSO THE
Doctrine of the *Centripetal Forces,*
AND
Velocities of Bodies,

Describing any of the
Conick Sections.

Being a Part of the
Great Mr. *Newton's* Principles.

THE
Whole Illustrated with Variety of
Useful THEOREMS and PROBLEMS,
AND
Accommodated to the Use of the Younger
MATHEMATICIANS.

By HUMPHRY DITTON.

LONDON:

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Books and Instruments. 1705.



C. L. e.

TO
The Very I N G E N I O U S
Charles Du-Bois, Esq;
OF THE
City of L O N D O N.

S I R,

A Person who comes upon the Account that I do ; to return Thanks for Kindnesses out of the common Road ; to have answer'd his Design the more effectually, shou'd have found some other Way for't than this. A Man seems but poorly to express his Sence of Favours that are altogether extraordinary ; when the Me-

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thod he takes to do it in, has the little Character of being a vulgar and customary One. Such is a Dedication : A Thing written *every Day* ; by *every Pen* ; and to *every Person* : The standing Compliment, that passes upon all Occasions ; and that which insolvent Debtors (those that wou'd be just) have ever made a Trade of Offering, instead of Payment. One therefore that is conscious of so great a Debt (as I wou'd be thought to own by this Address) must necessarily lie under a mighty Disadvantage. For neither great Obligations on the one Hand, nor the Acknowledgments of a grateful Mind on the other, can (by any Painting here) ever appear to be what they are. A Man may describe the Favours
of

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of a generous Friend, and his own just Sence and Resentments of them too : But after all ; be they ever so great in themselves, they lessen by coming down into *this Way of Representation*. Notwithstanding this, the *Great Law of Gratitude* urges me to obey it : And I have concluded, 'tis better to do it with some Disadvantages ; than to incur the Guilt of a total Neglect. Be pleased then (SIR) to let this stand as the Monument of a Gratitude that *wou'd have been great* : A small and trivial Acknowledgment I confess : But yet the genuine Product of that, which is allow'd to add a Value to the most despicable Things : For such is a sincere good Will. Had there been no Reasons on my part ; and I had

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consider'd only those that are to be drawn from your self; this Book had been inscribed to You of course. Every Subject that's fine and curious does of Right belong to You: But *Philosophical Matters* are peculiarly Yours. In the Midst of all Your weighty Busineses, You find Time to converse with Nature, to so good Purpose; that those that know You so well, as 'tis my Happiness to do, are sensible She's fond of Your Acquaintance; and does by very particular Favours bribe You to continue it. And tho' the Liberty I take in asserting the Truth may give some Umbrage to a *Vertue* (which at present shall be Nameless) yet so much do I confide in the Protection of Your *Temper*, that I dare venture to en-

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flame the Reckoning : I'll confess
the Crime to be voluntary and pre-
sumptuous, and that I knew before-
hand, how nice and tender You
were in this Point. If You (SIR)
think fit to carry on such a Design,
as that of hiding Beauties, which
ought to be exposed to the World :
You must pardon those People that
act so good a Part, as to discover the
Contrivance, and so are free as to
say, 'tis the only Peice of Injustice
You can be guilty of. Whatever
Vices, Addresses of this nature are
commonly chargeable with ; mine
to You, has Two Vertues to re-
commend it, Truth and Inno-
cence. For 'tis true in the high-
est Degree, that You have made
me Your Debtor ; as You do the
Rest of Mankind Your Admirers.

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Nor is it less true, that I do with
the greatest Pleasure subscribe my
self,

S I R,

Your most Obligated Friend,

And very Humble Servant,

H. DITTON.

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IN this little Treatise I have endeavour'd to explain that Part of Mr. Newton's Principia, which is Fundamental to the Rest of the Divine Book; and particularly to the System of the World. And I have all along taken Care to render Things so far plain and easie, that they may be understood by all those that will bestow some Application upon them, and have a competent Share of the necessary Furniture, namely Algebra and Conick Sections. 'Tis entirely owing to the stupendious Genius and Sagacity of this Author, that we have any such Theorems now extant in the World, as those which his incomparable Book contains. The Difficulties in the Theories of the Heavenly Bodies might have continued a lasting Reproach to the Mathematicks of our Earth, if such a Mind as his had not undertaken to vanquish and remove them.

The Materials that this Book is composed of, are so absolutely Mr. Newton's Property, that I dare hardly pretend to call any thing
mine.

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mine. The Principles most certainly are all his own: And if I have attempted any where to make any Uses of them, or to draw any Consequences from them; yet the indisputable Right that he has to the Former, gives him a Title to the Latter also, where they are just and good. This is certain, that his Inventions are new and compleat; and equally exclude all the Additions and Claims of those that come after. But that I may not be imagin'd to throw my Mistakes (if I have committed any) upon the Great Author, and by saying all is his, either be thought to reflect on him, or to design to screen, and shelter myself: Whatsoever is amiss and faulty, I take it entirely upon myself, and declare it not to be the Effect of the Principles that shou'd have guided me, but of my own Inadvertency too, and not being guided by them.

And as I ought not to have omitted (or said less than) this, upon this Head; so the strictest Justice, and the Rules of good Manners, as I apprehend the Matter, did both oblige me to say what Interest he had in this Book, and how many Ways 'tis his own: For tho' there's no manner of Danger in not owning Mr. Newton's Theorems; and his Inventions, in what Hand soever they are, will easily discover and point to their true Author; yet it seems at least handsome and descent for

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a Man to tell the World plainly when he takes any such Liberties.

As for the Reception this Undertaking is like to meet with in the World, I have no Uneasiness at all about the Matter. Some People cry down all Books of course, that are not writ by themselves, or those of their own Faction: As others do out of pure Spleen, because the ill-natur'd Subject won't stoop to their Understandings. Some People again naturally love to find Fault; their Genius prompts them to snarle and censure, and they take the same Pleasure and Satisfaction in that, that others do in the very best Employments of their Minds. In a Word, every Man that writes, and every thing that's written, runs the Risque of a Thousand Censures: As many at least, as there are Prejudices, Humours and Fancies to be met with. The Fates of good and bad Books are very often like those of good and bad Men in this World: So uncertain and confused, that the true Characters of either are often not to be known. In Writing as well as in Manners, he that does ill may chance to be commended for't, and he that does well without a just Recompence, ought to remember that many a good Thing has been serv'd so before.

*If any Person thinks this Work wou'd have become his own Hand better than mine; I'll easily yield that Point to him. For tho' a
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great many that make a Noise and a Bustle, can do little else but that: Yet there are some that will boast and perform too; so unlucky a Lodging have their good Qualities gotten. I desire only it may be minded, that my Design here, is not to entertain those that are of an advanced Standing in this noble Philosophy, but to introduce those that wou'd be acquainted with it, and have as much Mathematicks as is necessary to qualifie them for it. And therefore upon such an Account a Man is obliged to descend to a Multitude of little Things, that otherwise wou'd not be necessary, and may seem to take off from the Elegancy of what he does. 'Twou'd be an easie Matter, either by omitting at first, or afterwards taking off, several useful Superfluities brought in for the Sake of Beginners, to make Things of this Kind appear, in a more pleasing Dress, even to Judges.

Farther, To render what I have done more universally serviceable here at Home, I chose to make it appear in English rather than Latin. For if it be granted that Mr. Newton's Discoveries are but barely useful, there's no Reason why a Multitude of very capable Minds shou'd be debarr'd from them meerly for want of a Language. But if it be granted (as it will be by all that know them) that they are the finest Pieces of Human Knowledge

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in the World, then the Reason grows very strong, why they shou'd be put in such a Language as they understand. Again, a Man is bound to believe that the Illustrious Author design'd, what he was pleas'd to publish, shou'd be known; and that not to a few Persons only who fortunately had the Advantage of an Education; but to all whom Nature had qualified with good Reason, and their own Industry with Skill in Mathematicks. To think less than this, is to suppose him capable of envying a Part of Mankind the Knowledge of his Discoveries, which wou'd be no small Affront to a Soul much less than his. Now if there be any that shou'd say, 'tis to lessen and deprectate these sublime Pieces of Philosophy to expose them to the Vulgar by a Publication in English: That they ought to be entrusted only with those that have Skill and Judgment to make a right Use of them: And (in a Word) that others, by going away with crude imperfect Notions; will do themselves no Kindness, and disparage the Things they talk of into the Bargain.

Thus indeed, I confess, do some People argue for keeping the Sacred Books in an unknown Tongue: But we pretend to a Protestant Liberty, at least with respect to our Philosophy. And methinks both in the one and the other Case; 'tis unreasonable that those that wou'd
make

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make a good Improvement; shou'd be kept in Darkneſs, for the Sake of them, that wou'd abuſe their Light. If a curious Treatiſe ſhou'd not appear in the vulgar Tongue of a Country, for fear ſome People ſhou'd play the Fool with it, and make an impertinent Uſe of what it diſcovers: 'Twill be very hard to find what Language it ought to be printed in. For 'tis no unlikely thing that there ſhou'd be Fools of all Nations and Languages, and 'tis Ten to One but ſome of theſe, do meet with an Author, which ſoever of them all he chuſes. We vainly flatter our ſelves, no doubt, to think we have ſecured a fine Thing from vulgar Underſtandings, by putting it into Latin. For we ought to believe that there are Mechanicks in Learning, as well as in Trade, and ſome vulgar People that drudge with Books, as well as others with Spades and Hammers. 'Twere well if all Men cou'd get Skill in Languages; but good Senſe and this Skill are very often no Friends to each other, and won't live together in the ſame Head. And when they are apart, methinks, 'tis no hard Point to determine on which Side the vulgar Underſtanding lies. But to answer them that won't otherwiſe be answer'd; let them conſider in what Language our moſt celebrated Geometry Profeſſor, Capt. Halley, has condeſcended to publiſh a good part of his invaluable Performances. And let 'em conſider Mr. Newton's

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ton's Book of Light and Colours, where the rich Treasures lie expos'd also in our own Tongue.

And now I have done with that Matter: I shall only add as to the Book it self, that, I hope, the Perusal of it will neither be unpleasant nor unprofitable to the ingenious well qualified Reader. And when he comes to perceive the Use and Advantage of these Things, and how they tend to lead him into the most profound and pleasing Speculations of Nature; let him then return his Thanks to the great Genius to whom all this is owing; and if he thinks it fit to allow my Pains any little Share of his Acknowledgment, he'll make me his Debtor thereby, and bind me to farther Endeavours for his Service.

But tho' I have trespass'd upon the Reader's Patience thus long, I must beg his Pardon, if I desire the Continuance of it yet a little longer. The Laws deliver'd in this Treatise relating to the Motions, Forces and Velocities of Bodies, are but very particular and narrow Things, in respect of that great and universal Law of the World, that Motion is essential to Matter. The Person that advances this (and passes his Word for the Truth on't) is Mr. Toland, who has written a large Letter about it, which may be seen in his Book,

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entituled, Letters to Serena. And as all the great Laws of Nature are capable of many excellent and profitable Applications; so the Uses of this Principle are (if we dare believe him) both very many and very great. It's a Clue that will faithfully guide a Man thro' some of the most perplex'd Difficulties in Nature, and enable him to give such Solutions, as cannot be given any other Way. Mr. Toland says as much as this amounts to of it; which is the same Thing, as if he had said in other Words, That 'twill conduce to the making Philosophy as little mysterious as Christianity it self. How little that is, he, perhaps, very well knows; and how little Darkness this Notion leaves in one particular Point in Physicks, without anything of a Perhaps, I am sure I do know. For to give a clear and satisfactory Account of the Vis Motrix and the Vis Impressa; he says, That the Vis Motrix is the general and essential Action of Matter; and the Vis Impressa the particular Determination of that general Action. Now, this makes all the Mystery of the Business vanish at once, and the Philosophers need torture their Heads about it no longer. In a Word, other People may think as they please, but I must be excused, if I pretend to believe, that Mr. Toland is in Jest, and acts a Part. 'Tis much more civil to suppose this, I am sure, than to believe him to be in Earnest. For
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when a Man appears to be warm and serious in the Management of a Subject, talks a vast deal upon it, speaks with the last Degree of Assurance, and, after all, says nothing to the Purpose: To say, He believes what he says, is to give him the ill Name of a Fool, and to call him senseless and stupid; which are Terms I wou'd use to no Man. But to take that Part which I do, is at most but to make him a little Trickisbly disposed, and to love (in a merry Way) to put upon Peoples Understandings; which Character, perhaps, may seem less displeasing and disagreeable to Mr. Toland, than the former. In Confidence of this, I endeavour to justifie my Surmise by the following Reasons.

1. Because he takes particular Care to use no Argument, but such as he is sure not to conclude with: And if he does pretend to prove his Point, 'tis in such a Manner as can never possibly persvade the Reader to be of his Mind.

2. When he takes Notice of any thing that may be an Objection in his Way, he deals with it in a very Gentleman-like Manner; softly and tenderly, without making any rude Assault, or offering the least Violence to it.

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3. He is no less certain to baulk his Reader's Expectation, than he is careful to raise it: And after he has made very liberal Promises, does neither perform them, nor ask Pardon for the Omission.

4. He opposes very groundless precarious Notions to Truths that are sufficiently proved, and lays his own Opinions without any Reasons, in the Ballance with Truths that are capable of Demonstration.

And when a Man does such Things as these, 'tis not unreasonable to suspect, that he is either in Jest, or out of his Wits: But the first Supposition is the more good-natur'd and gentile of the Two, and therefore I rather make that than the other.

Let us now briefly attempt the Justification of these Reasons alledged. And in order to the first, we may observe, that this Gentleman expresses his Mind about Motion's being essential to Matter with the utmost Plainness and Perspicuity; so that 'tis impossible to doubt of or mistake his true Meaning. I hold, (says, he Pag. 159.) that Motion is essential to Matter; that is to say, as inseparable from its Nature as Impenetrability or Extension, and that it ought to make a
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Part of its Definition. *And* (Pag. 161.) My only Business is to prove Matter necessarily active, as well as extended. *And, to quote no more, he tells us* (Pag. 168.) He maintains, that Matter can be no more conceiv'd without Motion, than without Extension, and that the one is as inseparable from it as the other. *Now, after he has* (in Letter V.) *spent several Pages in discoursing upon the uncertain and changeable State of Matter every where in the Universe; shewing, that the Parts of it undergo a Thousand Vicissitudes and Alterations, and this in Bodies of all Forms and Textures, even those which we suspect the least liable to such Changes. Having done this, he comes to gather up all into a Conclusion, and infers after this manner* (Pag. 202.) I think, after all that has been said, I may now venture to conclude, that Action is essential to Matter; since it must be the real Subject of all those Modifications, which are call'd Local Motions, Changes, Differences or Diversities: And principally, because absolute Repose (on which the Inactivity or Lumpishness of Matter was built) is entirely destroy'd, and prov'd no where to exist. *Now all this is so perfectly Foreign to Mr. Toland's Purpose, that out of 1000 People that shou'd grant him the Truth of his Premises, 999 at least wou'd deny that his Conclusion*

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follow'd from it. Grant that the Parts of Matter every where throughout the Universe (tho' his Proofs do not pretend to reach so far) were in this State of continual Flux and Change. Grant that they undergo all the Removes and Shiftings of Place and Form that he speaks of, and Ten Thousand more. Let it be taken as a Truth, that there is no absolute Repose, but Motion in all Systems of Matter, both greater and less: And after all, I say, Quid Colligis inde? Will it follow, that this Action is of the Essence of Matter, and that Matter can't be Matter without it? By no Means in the World. Mr. Toland can't but know, that 'tis no good Arguing to say, this Thing is so in Fact, therefore 'tis naturally, necessarily, essentially so, and impossible to be otherwise. I find this Thing may be predicated of this Subject, and therefore the Subject can't exist, can't be defin'd, can't be conceiv'd of without it. If this were allowable, a Man might infer a Multitude of Things that are notoriously false and absurd, and which all Mankind will grant to be so, at the first Hearing. And therefore I wou'd hope I need not question but he was aware of it, and took this Method of proving only to shew, that, however he was pleas'd to banter, he did not believe what he said to be true.

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In the next Place, as to his Way of treating Objections, we shall find him ever curiously observing the Motions of a Difficulty towards him, and by prudent and well contriv'd Steps getting out of the Reach on't. For Instance, The Business of the different Specifick Gravities of Bodies equal in Bulk, being urged to prove a Vacuum, from whence the Inactivity of Matter wou'd be fairly inferr'd. Mr. Toland takes Notice of it (Pag. 183.) and as it deserves a Reply, so he bestows an Answer of Two whole Pages upon it. He returns, That a Man that urges this, must suppose Levity and Gravity not to be meer Relations; the Comparisons of certain Situations and external Pressures: But that they are real Beings, and absolute inherent Qualities, which is now by every Body exploded, and contrary to the Rules of Mechanicks. Here now is the Solutions: From whence he diverges immediately as far as the original Chaos it self, where he says, That there cou'd be no Gravity or Levity; and, That these Qualities wholly depend on the Constitution and Fabrick of the Universe. As for the Chaos it self, he affirms, The Notion of it is as confused and monstrous, as the Import of the Name; and built in every Step on Suppositions that are not only arbitrary, but utterly false and chimerical. Not that he

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wou'd be thought to affront Moses, or derogate any thing from the Scripture, which speaks of a Chaos exprestly; but only coming thus far, to be at the greater Distance from a Vacuum; he added this by way of Ecucidation and Enlargement. Now, had he had any real Spite at the Argument, he wou'd not have gone to telling a Tale of this, and that, and the other Thing that a Man must suppose that urged it; for that's meer Trifling: But presently endeavour'd to have shewn, how this Fact, which is plain and undeniable, viz. That Bodies of equal Magnitude have different Specifick Gravities; how this, I say, either had not been sufficient to infer a Vacuity; or else how his Doctrine of the Essential Activity of Matter might notwithstanding have been maintain'd. This had been fair and close to the Purpose, whereas what he does is very remote and far from it. 'Tis not a Jot to the Matter in Hand (as to the main Result) what Gravity is, or what it depends upon; if it be true and certain in Nature and in Fact, that Bodies of equal Bulk are thus unequally heavy. For this will manifestly infer, that the Quantity of Matter in the One cannot be so great as that in the Other; neither is the Consequence to be avoided, but by running away from't, as he does.

Again;

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Again; As to the old Argument for Vacuity, that without this a Body mov'd by another wou'd find no Place nor Room to betake it self into. To this he answers (with his wonted Moderation towards all Objections;) For you, (says he to the Opponent, Pag.178.) to speak in this Manner, is not only to have the same gross Conceptions with your Farmers, but also to suppose the Points B and C, and all, or most of the Points about them, to be really fix'd and in absolute Repose. But then the main Spirit of the Answer is behind; for he tells him, That if he succeeds in proving the natural, essential, intrinsick and necessary Motion of Matter, then all those Difficulties will vanish, and appear to be nothing. Which is as much as to say, He'll let the Argument alone for this time, but the next time he takes it in hand he intends it shall not come off upon so easie Terms. So that he fairly discharges it for the present, and bids it expect the Result of his having prov'd Motion essential to Matter. And perhaps this is more Civility than cou'd have been expected from any Man but Mr. Toland. Another Person that had confronted an Objection in this Manner wou'd hardly have parted with it without making one Attack at least; whereas he contents himself with bestowing a small Threatning upon it, and so lets it go without any farther Harm.

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But farther yet : 'Tis worth taking Notice how little Design to satisfy, there appears, in his Method of illustrating Difficulties, as well as answering Objections. Now his Way of doing this is, to make the very Expedient that is to remove one Difficulty, involve Two or Three more that are equally puzzling. For Instance (Pag. 176.) he tells us, That to believe the Activity of Matter will clear up all dark and doubtful Points about *Avacuum*. For (says he) as those particular or limited Quantities which we call such or such Bodies, are but several Modifications of the general Extension of Matter in which they are all contain'd, and which they neither increase nor diminish ; so all the particular Local Motions of Matter are but the several Determinations of its general Action, directing it this or that Way, by these or those Causes, in this or that Manner, without giving it any Augmentation or Diminution. Now were Mr. Toland very serious, an intelligent Reader wou'd think he design'd to affront his Understanding by offering such a Thing as this to him for the Solution of a Difficulty. And who cou'd do less, to see a Man cunningly top a Notion upon him (while he pretends to inform him) that is so absolutely precarious and obscure, as his Notion of particular Bodies and their Motions ?

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tions? They are the Modifications of the general Extension, and the general Action of Matter. But how is this made out? Why Mr. Toland says 'tis so: But if Mr. Toland shou'd swear 'tis so, and curse all Mankind into the Bargain that won't believe him, 'twill never pass for a Truth, for all that, amongst People that are guided by Reason. Ought not he to have put it past Dispute, that particular Bodies were no more than so many Modifications of the general Extension of the Universe, before he had assum'd it to solve a Difficulty withal? Or if he had done it any where afterwards, it wou'd have been sufficient. But never to do it, and yet to suppose it, is to beg his Point in the most sordid Manner imaginable, and to trifle with his Reader unpardonably, if he were not in Jest.

I know well enough that the Notion of Infinite Matter is a Darling with him, and that he may possibly trust to this, for the Proof of this Definition of particular Bodies. But I also know well enough that the Notion of Infinite Matter may be infinitely false and absurd, for any thing he has offer'd to prove the Contrary. And whereas he seems to be trying at it (Pag. 213, &c.) 'tis evident to any one (that can see when a Man presses on an Argument with Courage, and when he does it timorously) by the Doubtfulness of his Proceeding,

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ceeding, how much he suspected the Infirmary of his Reasoning upon that Head. However, there needs no more to be said; for (Page 215.) he sets the Infiniteness of Matter upon the same Foot with the Essential Activity of it, and he before took Care to offer no more than a Paralogism for the Proof of that Doctrine.

In the next Place; What Performances he gives his Reader Encouragement to expect from him, may be seen at Pag. 159. where he speaks of no less Things, than accounting for the same Quantity of Matter in the Universe, proving that there can be no Vacuum, solving all the Difficulties about the Vis Motrix, with a great many more besides these. And all this from the Principle of the Essential Activity of Matter: Nay more than that, he affirms that this Principle alone can do it. But if any impartial Reader can find, that he has either evinced the Essential Activity of Matter; or that he has fairly accounted for these Difficulties, or any One of the whole Number; or that he has (much more) shewn that no other Principle can do it, but that alone; I'll then beg his Pardon for asserting that he has not. In the mean time, I can't transcribe his whole Book, but every Reader that will take the Pains may see whether I am in the Right or no.

Lastly,

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Lastly; He makes no Scruple of setting any of his own unprov'd precarious Opinions, in Competition with Things that have convincing Proof and Evidence of their side, and are universally own'd to be such. 'Tis a Point that Mr. Newton has set past all Dispute, that the Weights or Gravitating Forces of Bodies at equal Distances from the Center, are proportional to the Quantity of Matter in them. But with Mr. Toland this is yet a Controversie ---- Least I engage my self in a Dispute (says he) about the Nature of Gravity, as whether the Weight of Bodies be proportional to the Quantity of Matter, &c. Pag. 207.

Again; 'Tis a Truth sufficiently clear, that the external Pressures of the Atmosphere and Aether, have nothing to do, as causes either in whole or in part of such an Effect as Gravity in Bodies; therefore external Pressures can be of no Significancy to the Producing such an Effect as the different Specifick Gravities of Bodies. But he is pleased to declare his Opinion to be otherwise, and allow Pressure a Share in that Phenomenon.----- Tho' their Specifick Gravities (speaking of Cork, Lead, Gold, and the like) be so different from each other; proceeding partly from external Pressures, and partly from those internal Structures and Modifications, which give their common Matter those various
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Forms that constitute their Species, &c. Ibid. This (says he) is my own Opinion, whatever be my Reasons for it. Which is admirably well said; and perhaps he knew well enough why he shou'd keep his Reasons to himself. But however; this very Passage alone is enough to justify my Surmise, that Mr. Toland wou'd be thought to banter. For to offer an Opinion that thwarts a Matter of Demonstration, and say, I believe thus and thus whatever be my Reasons for't, is almost the same Thing as to say, I am in Jest.

In short, Mr. Toland appears to me, to manage the Point much after the same Manner with Lord Peter in the Tale of a Tub. The One calls a brown Crust, Mutton and Claret; as the Other does Fancy and Paralogism, Reason and sound Philosophy. Both appear grave and serious in what they do and say, and agree in acting an imposing Part; One upon the Sences, the Other upon the Understandings of those they talk to. The Difference is, that One swears to the Truth of what he says, and the Other is so modest as to pawn his Word for't. Tho' there's no question but Peter might have some Reasons too lying by him, to prove what he said to be true: Only (like this Gentleman) he was not pleased to produce them. 'Twas sufficient for him to say, 'Tis my Resolution, that this

which

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which looks like a brown Crust, is really what I have told you, whatever be my Reasons for't. But Peter's Mutton and Claret was certainly never more like a Piece of a brown Loaf, than Mr. Toland's Philosophy is like Paralogism. The Resemblance is such, that I doubt not, he has or will find, 'tis universally taken for the very same Thing in the World.

I shall say no more; but only that I hear, some Questions will shortly be proposed to him upon the Business of Real Space. For my own Part I have nothing to do with that Point, and as much as I value the Judgments of some that defend it, in other Matters, cannot subscribe to them in this. I do believe, as well as Mr. Toland, that many of those Gentlemen firmly believ'd the Existence of a Deity. But I also believe too, that Infinite necessarily-self-moving Matter may serve to entertain an Atheist, as well as Almighty Space.

It remains only now, for me to return my humble Thanks to the Reader, and bid him, Farewell.

N.B.

N. B. The Reader may, if he pleases, omit the *Lemma* (at *Pag.* 138.) and take the whole Process (for the Investigation of the following *Theor.* 4.) thus. Using all the Symbols as in the *Lemma*, from the Nature of the Circle 'tis $YY = 2RX - XX$, and $yy = 2rx - xx$; also $D^2 = 2RX$, and $d^2 = 2rx$. But at least D and d are coincident with A and a , and X , x with $\dot{X}$, $\dot{x}$; so that then we have $A^2 = 2R\dot{X}$, and $a^2 = 2r\dot{x}$, therefore $\dot{X} : \dot{x} :: \frac{A^2}{2R} : \frac{a^2}{2r}$. Q. E. I. In this manner, I say, the Reader may come to the *Theorem*, without the Assistance of the *Lemma*, which gives the last *Ratio* of the Subtense of the Angle of Contact to the conterminal Arch. Tho' the *Lemma* it self too be contain'd in the Process; for since $A^2 = 2R\dot{X}$, and $a^2 = 2r\dot{x}$, therefore if $2R = 2r$, then $\dot{X} : \dot{x} :: A^2 : a^2$; as Directions are given for the Reading of it in the *Errata*.

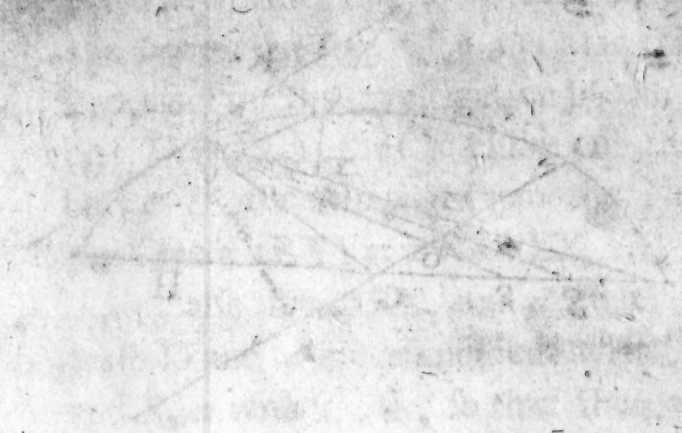


Fig. 20.



...is ever equal and contrary to A...
 ...of the Actions of Two Bodies upon one
 ...are always equal, and have contrary

OF THE

LAWS of MOTION,

WITH THEIR

Application to *Mechanicks*.

PART I.

A

LAW I.

ALL Bodies will persevere in their State of Rest, or uniform direct Motion, unless they are compell'd to change that State by some Force impress'd upon them.

LAW II.

The Alteration of Motion is ever proportional to the impress'd Force (that causes it,) and the Direction of it is in the same Right Line with that, according to which the Force is impress'd.

B

LAW

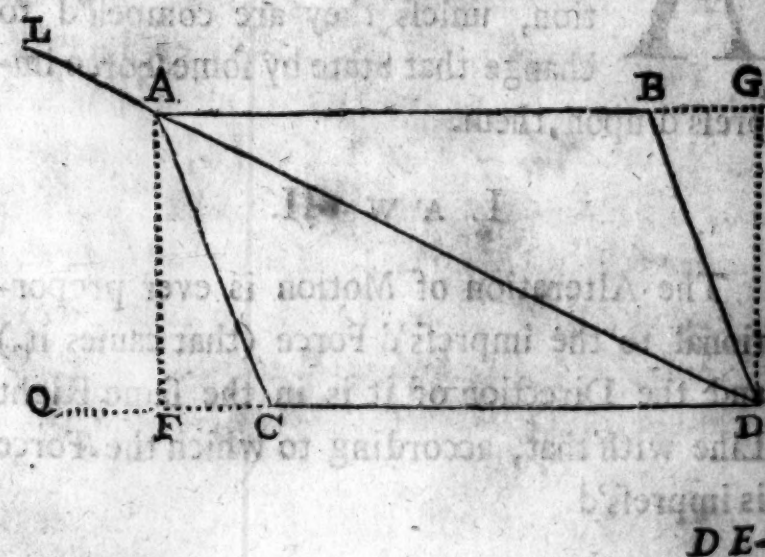
L A W III.

Reaction is ever equal and contrary to Action; or the Actions of Two Bodies upon one another are always equal, and have contrary Directions.

From these Laws the Illustrious Author deduces the following Corollaries.

First COROLLARY to the Laws of Motion.

A Body will describe the Diagonal of a Parallelogram by Two several Forces combin'd, in the same Time that it would describe the one or the other of those Sides by the one or the other of those Forces separately consider'd.



D E M O N S T R A T I O N.

Suppose a Body could describe the Line AB (FIG. I.) in the Time T, with the uniform Motion M; and in the same Time, the Line AC, with the uniform Motion N. Compleat the Parallelogram, and draw the Diagonal AD. The Diagonal AD shall be described by these Two Forces thus combin'd in the same Time T: For the Force N acting with the Direction AC, parallel to BD, cannot change the Velocity of the Motion towards BD; the Body therefore will be carry'd to the Line BD in the same Time T, whether the Force N be impress'd or no; therefore at the End of the Time T it will be found somewhere in the Line BD. In like manner, the Force M acting with the Direction AB, parallel to CD, cannot change the Velocity of the Motion towards CD: Therefore the Body will be carry'd to the Line CD, in the same Time T, whether the Force M be impress'd or no; consequently at the End of the Time T it will be found somewhere in the Line CD. Therefore since at the End of that Time it will be found in both these Lines BD and CD, it will certainly be found in D their Intersection, that is in the Diagonal Q. E. D.

From hence arise the following Theorems.

T H E O R. I.

The Forces by which the Sides and Diagonal of a Parallelogram are described, are proportional to, and may consequently be expounded by those Sides, and that Diagonal respectively. For the Motions being uniform by the Hypothesis, and the Times of the Description (both of the Sides and Diagonal) being prov'd to be the same by the foregoing Demonstration, the Spaces described shall be as the Velocities by *Law II.* or as the Forces to which these Velocities are proportional respectively. Therefore (putting the Force in $AD = G$) it will be $G. M :: AD. AB$ and $G. N :: AD. AC$ and $G. M + N :: AD. AB + AC$.

T H E O R. II.

If a Body with a Force proportional to AD describes the Right Line AD , the Motion, and Direction of that Motion will be the same as if the Body had at first been impell'd by Two Forces, acting according to the Directions AB and AC , and proportional to those Lines. Therefore any Motion, tho' in it self ever so simple, may be considered as compounded of others, and (from this Principle) may be resolved into them.

T H E O R.

T H E O R. III.

Suppose the Parallelogram as before, and a Body to be held immovable in the Place A by Two equal Forces acting with contrary Directions; the one tending from A to D, and the other from A to L. I say 'tis the same thing as if it were held by Three Forces acting with the Directions from A to L, from A to B, and from A to C. For the Force AD (by *Theor. II.*) is equivalent to the Forces AB and AC. Therefore, if the Body be held immovable by the Forces, whose Directions are AD and AL, it will be held by the Forces acting with the Directions AL, AB and AC.

T H E O R. IV.

A Body being thus held immovable by Three Powers; those Powers are one to another directly as the Lines that are drawn parallel to their respective Directions, and terminated at the Point of their mutual Concourse. For (by *Theor. I.*) the Powers acting with the Directions AD, AB, AC are in Proportion as those Lines AD, AB, AC respectively taken; that is, as the Lines parallel to the Directions, and terminated at their common Intersection.

T H E O R. V.

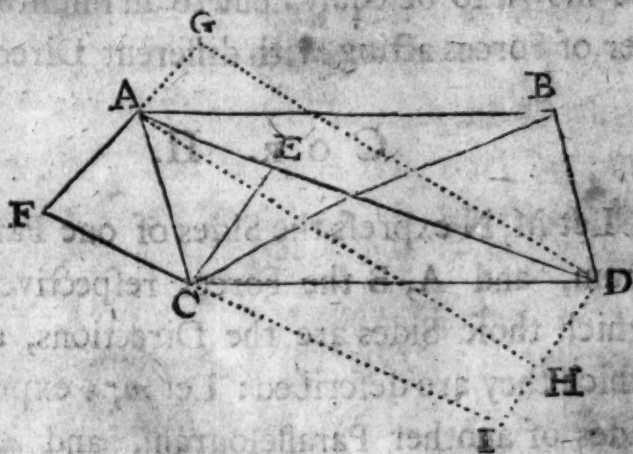
Therefore any Two of these Powers are to one another reciprocally as the Sines of the Angles which their Lines of Direction make with the Line of Direction of the Third Power. Thus the Power M (whose Direction is AB) is to Power N (whose Direction is AC) as AB to AC or BD, that is (because in any Triangle the Sides are proportional to the Sines of the opposite Angles) as the Sine of the Angle ADB or CAD, to the Sine of the Angle DAB. Now CAD is the Angle which the Line of Direction of the Power N makes with AD the Line of Direction of the Third Power; and DAB is the Angle which the Line of Direction of the Power M makes with the same Line AD: Therefore the Powers are as the Sines of those Angles reciprocally. Thus also the Power G (whose Direction is AD) is to the Power M; as the Sine of the Angle BAC, is to the Sine of the Angle DAC; that is, as the Sine of the Angle ACQ, or the Sine of the Angle ACD (the Complement of the Former to the Two Right Angles) is to the Sine of the Angle DAC; that is, as AD to CD.

General

General COROLLARIES.

C O R. I.

A Force acting with any given Direction may be shewn to be equivalent to an infinite Number of Forces acting with different Directions. For one and the same Line may be the Diagonal of an infinite Number of Parallelograms. Thus the Force in AD in the Paral-



lelogram (*FIG. II.*) ABCD is equivalent to the Forces in AB and AC; and in the Parallelogram AGDH the same Force is equivalent to the Forces in AG and GD, and so of infinite others. Also since the Sides of the Parallelogram ABCD may be made the Diagonals of an infinite Number of others, the Force in CB may be shown to be equivalent to an infinite Number of other Forces. For Example; Make the Side AC the Diagonal of the Parallelogram FAEC, and then

the Force acting according to the Direction CB will be equivalent to the Forces acting according to the Directions CF, CE and CD; and if CD be made the Diagonal of the Parallelogram CEDI, the same Force in CB will be equivalent to the Forces acting in CF, CI, and double the Force according to the Direction CE, and so continuing to make the Sides of these last Parallelograms the Diagonals of others, and so on infinitely, the Force in CB may be shown to be equivalent to an infinite Number of Forces acting with different Directions.

C O R. II.

Let M, N express the Sides of one Parallelogram, and A, B the Forces respectively, of which those Sides are the Directions, and by which they are described: Let m, n express the Sides of another Parallelogram, and a, b the Forces. Also let the Diagonal common to both be D, and the Force whose Direction is in that Line be F. Then (by *Theor. I.*) $A+B. F :: M+N. D$, and $a+b. F :: m+n. D$. Therefore $A+B : a+b :: M+N : m+n$. That is, the Sums of the Forces in the Sides of the Two Parallelograms, are directly as the Sums of the Sides.—Likewise (from the foremention'd *Theorem*, because $A : F :: M : D$, and $B : F :: N : D$. and consequently $A \times D = F \times M$, and $B \times D = F \times N$; there-

therefore $A \times D - B \times D = F \times M - F \times N$; that is
 $A - B : F :: M - N : D$. And so for the other
 Parallelogram, it may be shewn, that $a - b : F ::$
 $m - n : D$. Therefore (by Equality of Proportion)
 $A - B : a - b :: M - N : m - n$. That is, the Dif-
 ferences of the Forces in the Sides of the Two
 Parallelograms, are directly as the Differences
 of the Sides.

C O R. III.

Let the former Symbols stand as before ;
 only let m, n , now express the Sides of a Paral-
 lelogram, whose Diagonal is unequal to that
 of the former ; and let this Diagonal be put
 $= d$, and the Force by which 'tis described be
 put $= f$. Also let it be $D : d :: F : f$. Then (by
Theor. I.) $A + B : F :: M + N : D$, and $a + b :$
 $f :: m + n : d$; therefore $A + B : a + b :: M + N :$
 $m + n$. It may also be proved in like manner,
 as in *Cor. II.* that $A - B : a - b :: M - N : m - n$.
 So that tho' the Parallelograms have unequal
 Diagonals, yet those Diagonals being described
 by uniform Forces proportional to the Lengths
 of them ; the Sums or Differences of the Forces,
 that describe the Sides of those Parallelograms,
 shall be directly proportional to the Sums or
 Differences of the Sides themselves.

C O R. IV.

In every Rectangular Parallelogram, the Force in the Diagonal is equal in Power to the Forces in the Sides. This is *Theor. II. of Galileus's 4th Mechanick Dialogue.*

The following Problems relating to this Matter may here not improperly be subjoin'd.

P R O B. I.

To find Parallelograms having a common Diagonal, in which the Sums or Differences of the Forces (by which the Sides are describ'd) shall still be equal to one another, or be express'd by a standing Quantity.

According to the Tenour of *Cor. II.* let the Quantity $M+N$ be the Transverse Ax of any Ellipse, and D be equal to the double Excentricity, or to the Distance of the Foci from one another. Also let M, N be Two Lines drawn from the Foci to any one and the same Point in the Curve, and m, n Lines drawn from the Foci to any other Point in the Curve. I say, that the Lines M, N , and m, n , are the Sides, and that D is the common Diagonal of Two such Parallelograms as are required. For from the Nature of the Ellipse, $M+N$ is $=m+n$ perpetually; and therefore $A+B=a+b$ ever also, by *Cor. II. Q. E. I.*

If

If the Differences of the Forces were required to be a standing Quantity, and such Parallelograms were to be determin'd. Then let $M-N$ = the Transverse Ax of any Hyperbola, and D = the Distance of the Foci. Then from the Nature of this Curve, $M-N$ shall $=m-n$, and consequently $A-B=n-b$ perpetually.

P R O B. II.

To find Parallelograms having unequal Diagonals, in which the Sums or Differences of the Forces, with which the Sides are described, shall still be equal, or be express'd by a standing Quantity.

According to the Tenour of *Coroll. III.* let $M+N$ be the common Transverse Ax of any Two Ellipses, in which D and d are the Distances of the Foci; D for one Ellipse, and d for the other. Then if M, N , and m, n express Lines drawn from the Foci to any Points in the Two Curves (that is, M, N from the Foci of the one Ellipse to any Point in that Curve, and m, n from the Foci of the other Ellipse to any Point in that Curve) I say, that these are the Sides, and the Focal Distances the Diagonals of such Parallelograms as are required. For since $M+N$ is $=m+n$ perpetually (from the Nature of the Ellipse) therefore also, according to the Intent of that 3d *Cor.* $A+B$ shall be ever

=

$= a+b$. Q: E: I. So if M—N were the common Transverse of Two Hyperbola's, in which the Distances of the Foci were D and d , then M, N, and m, n being Lines from the Foci, as before; the Parallelograms will be such as is required: And M—N being ever $= m—n$ from the Genius of the Hyperbola, A—B shall $= a—b$; that is, the Differences of the Forces shall be always equal.

P R O B. III.

To find Parallelograms having either equal or unequal Diagonals, in which the Sums or Differences of the Forces with which the Sides are describ'd, shall still be in any *Ratio* assign'd.

Let the *Ratio* assign'd be that of $p. q.$

C A S E I.

Let the Parallelograms be requir'd to have equal Diagonals. Put the Transverse Ax of any Ellipse or Hyperbola $= T$, the Parameter $= L$, the Transverse Ax of any other Ellipse or Hyperbola $= t$, and the Parameter $= l$. 'Tis manifest, that in order to the satisfying the Demands of the Problem, the Ellipses or Hyperbola's must be so proportion'd to each other, that their Transverse Axes may be in the *Ratio* assign'd, and their Focal Distances equal. One Ellipse or Hyperbola therefore being given, whose

or

Transverse and Parameter, T and L , are known :

I say, that another Ellipse whose Transverse

Ax t is $= \frac{Tq}{p}$, and its Parameter $l =$

$\frac{Tq^2 - Tp^2 + Lp^2}{p}$; or another Hyperbola,

whose Transverse $= \frac{Tq}{p}$, and its Parameter $=$

$\frac{Tp^2 + Lp^2 - Tq^2}{p}$; these 2 Ellipses, or these 2

Hyperbola's shall have the Conditions required,

and the Parallelograms (whose Diagonals are

the Distances of the Foci, and whose Sides are

the Lines drawn from the Foci to any Points in

the Curves) shall be such as will answer the

Demands of the Problem. The Demonstration

of which is easie: For since the Square of the

Distance of the Focus from the Center, is in

the Ellipse $=$ to the Difference, and in the Hy-

perbola $=$ to the Sum of the Squares of the

Semitransverse and Semiconjugate; therefore in

the Two Ellipses or Hyperbola's, if the Di-

stances of the Focus from the Center in one be

put equal to that in another, we have this

Equation $T^2 - LT = t^2 - lt$ for the Ellipses,

and $T^2 + LT = t^2 + lt$ for the Hyperbola's.

From whence L and T being given in the one,

and also t in the other, the Parameter l of that

other may be found, and the Value of it will

be such as is determined above. For because

t is $= \frac{Tq}{p}$, and consequently Given; therefore

the

the Value of l will be express'd all in the Terms of the Given Quantities L, T, p, q . Lastly; Because the Transverse Axes are in the Given *Ratio* of p to q , by the Hypothesis; and the Sums or Differences of the Lines from the Foci to the Curves are equal to the Transverse Axes from the Nature of these Sections, and therefore ever in the same *Ratio* of $p : q$; and because (by *Cor. II.*) the Sums or Differences of the Forces are ever proportional to the Sums or Differences of the Sides (which are the Lines drawn from the Foci to the Curves:) Therefore the Sums or Differences of the Forces shall always be in the Given *Ratio* of $p : q$. $Q : E : D$.

C A S E II.

Let the Diagonals be required to be unequal. If T be the Transverse of one Ellipse or Hyperbola, then $\frac{Tq}{p}$ shall be that of the other. And if the Distance of the Foci in one Ellipse or Hyperbola be D , and that in the other be d ; these being the Diagonals, and the Lines drawn from the Foci to the Curves being the Sides of the Parallelograms: If the Diagonals be describ'd by Forces proportional to their Lengths, the Parallelograms thus made shall be such as are required. The 3d *Corollary* (with those so often mention'd Properties of the *Conick Sections*) demonstrate this without any further Trouble.

P R O B.

Diagonal CF, making an Angle as GCF with the former. Put $AC=z$. $CF=v$. $BC=x$. $AB=d$. $FD=b$. $DB=c$. Therefore $DC=c-x$.

Note, That the Points A and B are given; therefore AB is a given Quantity. So because F and D are given, the Line FD is ever the same; and likewise DB is invariable. As for all the rest ('tis evident) they are flowing Quantities. Let m be the Time in which AC is described, and r the Velocity in the same, n the Time in which CF is described, and s the Velocity; which Velocities I suppose to be the same in all the Points of the Lines AC and CF respectively, and consequently the Quantities r and s to be standing ones. From the common Principles of Mechanicks, 'tis evident, that $m.n :: z \times s. r \times v$. That is, the Times are in the *Ratio* compounded of the direct *Ratio* of the Spaces, and reciprocal *Ratio* of the Velocities. Also, if m, n , express'd Forces, and r, s , Resistances, then the Forces wou'd be in the *Ratio* compounded of the direct *Ratio* of the Spaces, and the direct *Ratio* of the Resistances. Therefore if s denotes the Resistance in AC or z , and r that in CF or v ; then one and the same Expression shall serve with Respect to both these Ways of proposing the Problem. Therefore (in one Supposition) the Time in which the whole Space ACF is described (or the Sum of the Forces in the other) will be as $zs + rv$.

But

But from the Rectangular Triangles, we have

$$z = \sqrt{xx + dd}^{\frac{1}{2}} \text{ and } v = \sqrt{bb + cc - 2cx + xx}^{\frac{1}{2}}; \text{ Therefore the Expression comes to this } s = \sqrt{xx + dd}^{\frac{1}{2}}$$

$+ r \sqrt{bb + cc - 2cx + xx}^{\frac{1}{2}}; \text{ which Expression is to be determin'd to a Minimum. Therefore working by the admirable Method of Fluxions, we shall find } s \times \dot{x} \dot{x} + r \times \dot{x} \dot{x} - \dot{c} \dot{x} = 0$

$$\frac{xx + dd}{2} \dot{x} + \frac{bb + cc - 2cx + xx}{2} \dot{x} - \dot{c} x = 0$$

than is, $s \times x = r \times c - x$. From whence $xc - x ::$

$$rx, s \times v. \text{ Therefore if the Points A. and F}$$

be supposed to lie in the Circumference of a

Circle of which C is the Center; that is, if

AC = CF, then it will be $x : c :: r : s$. And

consequently if the Line DB be divided in C,

so that BC : DC, as the Velocities in the Dia-

gonals AC, CF directly, or as the Resistances

reciprocally; that is, if the Sines of the In-

clinations of the Diagonals AC, CF to the per-

pendicular ICK, are in either of these Ratio's,

then the inflex'd Path ACF shall be described in

the least Time, of all that are in like manner

comprehended between any Two Lines drawn

from the Points A, F, to any Point in the

Line DB, or (with Respect to the other Way

of proposing the Problem) the Sum of the

Forces with which the Two Diagonals AC,

CF are described, shall be the least possible.

Q. E. I.

AM

C

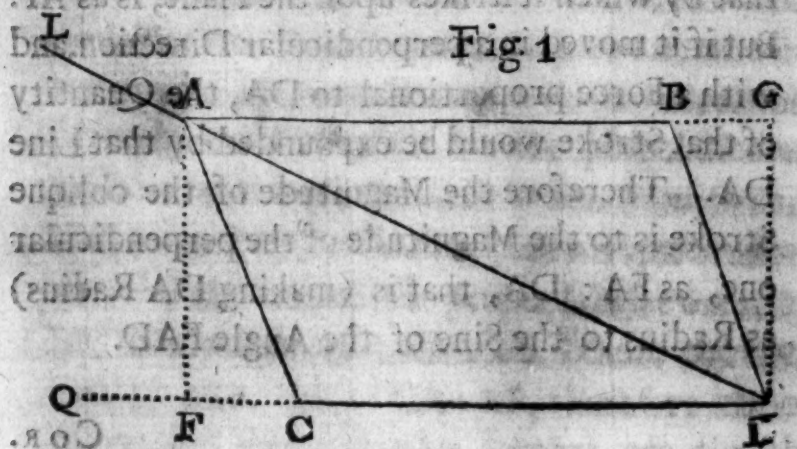
To

To carry this on a little farther ; If $r=s$, that is, if (the Body meeting with nothing at C to retard its Motion) the Velocities in AC and CF be the same ; then $x. c-x :: z. v$. In which Case EC will be coincident with LC, which makes one Right Line with AC. Also LH being perpendicular to BD produced, the Point D becomes now coincident with H. The Triangles ABC, LCH therefore are similar. For BC being $=x$ (as before) CH is now $c-x$, and BH $=c$. But 'tis found, that $x. c-x :: z. v$. that is, BC. CH :: AC. CL, and the Angles ACB and LCH are equal ; therefore, &c. From hence making HE = LH, the Triangle HEC will be equal and similar to HLC, and also similar to ACB. Now if the Body (moving in the Direction AC) instead of passing on in the Right Line CL, were reflected at the Point C, then since the Velocity continues still the same (the Direction only being changed) in the same time that it would describe CL, if it went on directly, it will describe a Right Line $=$ to CL in its reflex'd Motion. But from the Equality and Similarity of the Triangles HLC and HEC the Line CE is $=$ CL, therefore it will describe CE. But the Angle HCE is $=$ (HCL $=$) ACB ; therefore the Body will be reflected so, that the Angles of Incidence and Reflection (ACI and ECI) shall be equal.

N. B. I abstract here from all the Imperfections of Matter, and suppose the Body perfectly Elastick, and the Obex firm.

S C H O L.

If AC were supposed to be a Ray of Light, the Theorems found would be those Two celebrated ones in *Dioptricks* and *Catoptricks*, upon which those Sciences are built. And DC, CB, which are the Sines of the Inclinations of the Diagonals FC, AC, to the perpendicular ICK; would be the Sine of the refracted Angle, and the Sine of the Angle of Incidence, if DCB were the Surface of a different *Medium*. From whence those Sines would be as the Velocities in the Two *Mediums*, by what was found above, and as Mr. *Newton* has shown *Prop. 95. Lib. 2.* But this only in *Transitu*. The general Principle concerning the Composition of Forces shall now be apply'd to several Problems in *Mechanicks*.

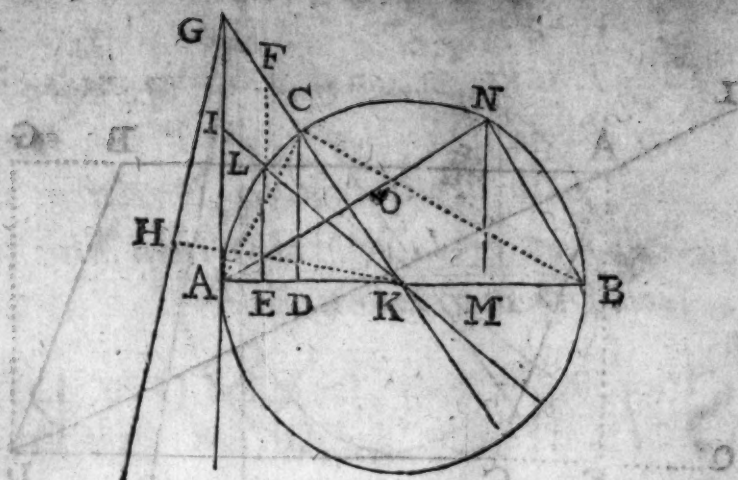


P R O B. I.

To assign the Proportion of a Stroke made with any oblique Direction upon a Plane, to one made with a perpendicular Direction, the Body moving in both Directions with the same Degree of Velocity.

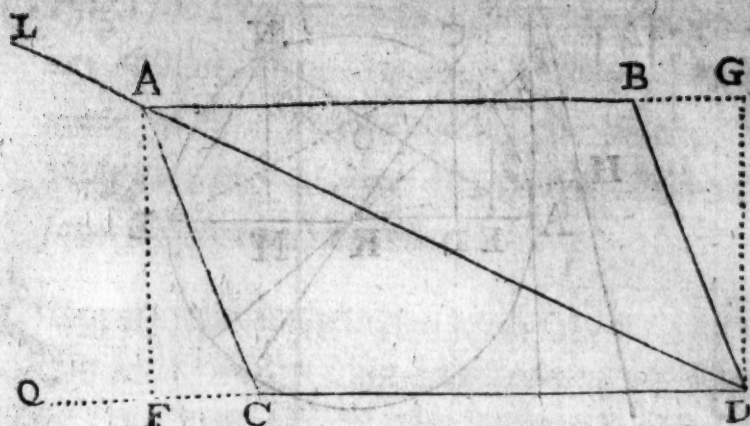
SUPPOSE the Body's Direction (*FIG. I.*) to be in the Right Line *AD*, and that it struck upon the Plane *DF* at the Point *D*. Let fall *AF* perpendicular to *FD*, and compleat the Parallelogram *FDAG*. The Motion of the Body in the Line *DA* is equivalent to Two other Motions whose Directions are *AF* and *AG*, by *Theor. II.* But the Motion whose Direction is *AG* is of no Significancy as to the Stroke upon the Plane *DF*, for *GA* being parallel to *DF*, the Body moving with that Direction would never meet the Plane. Therefore the Force by which the Body moves in *AD*, being as *AD*; that by which it strikes upon the Plane, is as *AF*. But if it moved in a perpendicular Direction, and with a Force proportional to *DA*, the Quantity of that Stroke would be expounded by that Line *DA*. Therefore the Magnitude of the oblique Stroke is to the Magnitude of the perpendicular one, as *FA : DA*, that is (making *DA* Radius) as Radius to the Sine of the Angle *FAD*.

COR.



C O R. I.

The Angles of Obliquity upon the Planes being equal, the oblique Strokes are directly proportional to the perpendicular ones, how unequal soever the Velocities (with which the perpendicular Strokes are made) be. Thus in the Rightangled Triangle ACB (*FIG. IV.*) the perpendicular CD being let fall; since $ABC = ACD$, therefore the oblique Stroke with the Direction AB upon the Plane BC, is to the perpendicular Stroke made with the same Velocity, as the oblique Stroke with the Direction AC upon the Plane CD, to the perpendicular Stroke made with the same Velocity with this oblique one. For (by the *Prop.*) the oblique Strokes are to their respective perpendicular ones, as $CA : AB$, and as $AD : CA$; but (from the Elements) $AD : CA :: CA : AB$, therefore the oblique and perpendicular Strokes are directly proportional.



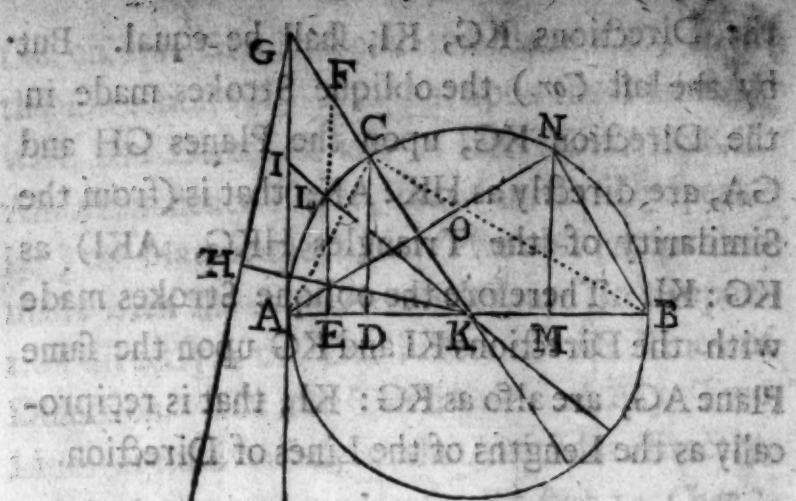
C O R. II.

The oblique Stroke made upon the Sides of any Rectangle, with equal Velocities, and with the Direction of the Diagonal, do answer reciprocally to the Forces that describe those Sides, and by the Composition of which the Diagonal is describ'd. For (*FIG. I.*) the Quantity of the perpendicular Stroke upon the Sides AF, AG, being expounded by the Diagonal DA, the oblique ones made upon the same Sides with the same Velocity, and the Direction DA, shall be as FD and DG. The Force by which the Diagonal is described being expounded by DA, the Forces in the Sides AF, AG shall be as DG and FD. So that the Magnitudes of the Strokes, and the Forces do answer reciprocally to one another.

C O R.



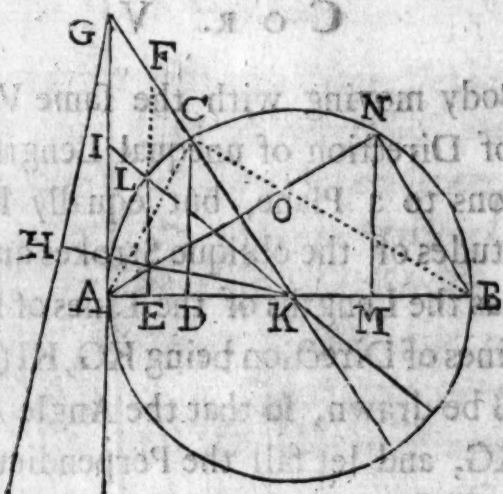
C O R,



C O R. V.

A Body moving with the same Velocity in Lines of Direction of unequal Length, and Inclinations to a Plane, but equally high; the Magnitudes of the oblique Strokes are reciprocally, as the Lengths of the Lines of Direction. The Lines of Direction being KG, KI (*FIG. IV.*) let HG be drawn, so that the Angle AGH may be = IKG, and let fall the Perpendiculars KA, KH as before. 'Tis evident, that the Angle KGH is = the Angle KIA. Therefore (by *Cor. I.*) since the Angles KGH and KIA are equal, the oblique Strokes made with the Directions KG and KI, upon the Planes HG and AI, shall be directly proportional to their respective perpendicular Strokes. Consequently, if the Body moves with the same Velocity, and so the perpendicular Strokes be equal, the oblique ones upon the Planes HG and AI, with the

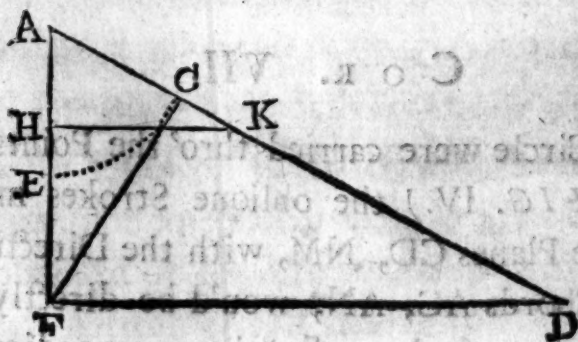
the Directions KG , KI , shall be equal. But by the last *Cor.*) the oblique Strokes made in the Direction KG , upon the Planes GH and GA , are directly as $HK : AK$, that is (from the Similarity of the Triangles HKG , AKI) as $KG : KI$. Therefore the oblique Strokes made with the Directions KI and KG upon the same Plane AG , are also as $KG : KI$; that is reciprocally as the Lengths of the Lines of Direction.



C O R. VI.

Therefore universally (joining the Accounts of *Cor.* III. and V. together) any oblique Strokes made by Lines of Direction neither of equal Lengths, nor equal Altitudes, are in the *Ratio* compounded of the direct *Ratio* of the perpendicular Altitudes, and the reciprocal *Ratio* of the Lengths of the Lines of Direction; *Ex. gr.*
the

Strokes upon the same Planes. For, with respect to the Plane CD, the Magnitude of the oblique Stroke, is to the Magnitude of the Perpendicular, as $AD : AC$. Likewise with respect to the Plane NM, the Proportion of the Strokes is as $AM : AN$. But from the Nature of the Circle, $AD : AM :: AC : AN$. Therefore, &c. With respect to the Planes CD and BC; the oblique Strokes made upon them, with the Directions AC, AB respectively, would be directly proportional to their correspondent perpendicular Ones. For the Triangles ACD, ABC are similar, and the Angles ACD, ABC (which the Lines of Direction make with the Planes) are equal : Therefore this Proportion of the Strokes holds all over the Circle. With respect to the Planes AN and BC, the oblique Strokes made with the Directions AO, and BO, are also directly proportional to their correspondent perpendicular Ones. For the same Reason as the last.



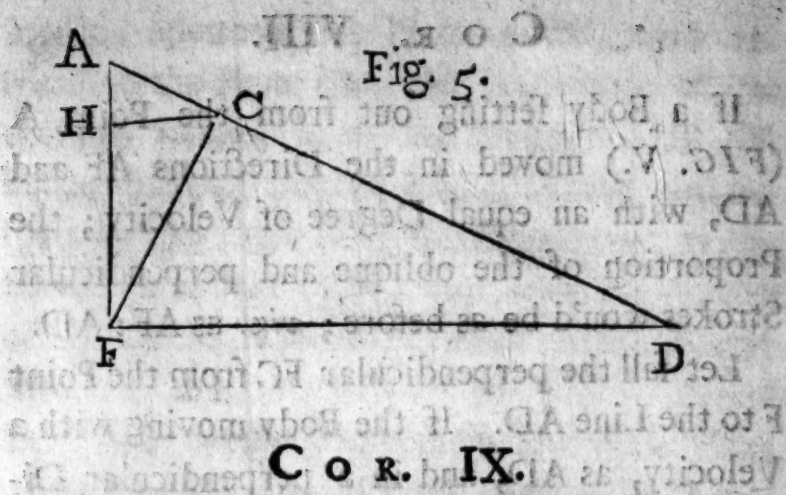
C O R.

C O R. VIII.

If a Body setting out from the Point A (*FIG. V.*) moved in the Directions AF and AD, with an equal Degree of Velocity; the Proportion of the oblique and perpendicular Strokes wou'd be as before; *viz.* as AF: AD.

Let fall the perpendicular FC from the Point F to the Line AD. If the Body moving with a Velocity, as AD, and in a perpendicular Direction, struck the Plane at the Point D, the Magnitude of the direct Stroke at that Point, wou'd be expounded by AD, and that of the oblique Stroke in the Direction AD, wou'd be expounded by AF. Likewise, if the Body moving with a Velocity, as AF, and in a perpendicular Direction, struck the Plane at D, the Magnitude of the direct Stroke at that Point would be expounded by AF, and that of the oblique one in the Direction AD, would be expounded by AC (by *Cor. I.*) for $AD: AF :: AF: AC$. But the Magnitude of the direct Stroke is the same, whether it be given at the Point D with a Velocity as AF, or at the Point F with a Velocity as AF. Therefore the Proportion of the Strokes remains the same.

COR.



C O R. IX.

The oblique Stroke (*FIG. V.*) made in the Direction *AD*, is equal to a direct Stroke (made with a Velocity as much less than that which the Body moves in *AD*; as *AF* is less than *AD*.)

For the oblique Stroke is to the direct one made with the same Velocity as *AF*: *AD*. Now any Two direct Strokes made by the same Body are one to another, as the Velocities with which the striking Body moves. Therefore if these Velocities be as the Lines *AF* and *AD*, then the Magnitude of the oblique Stroke with a Velocity as *AD*, is equal the Magnitude of the direct Stroke with a Velocity as *AF*: Or, which is all one, the Magnitude of the oblique Stroke with a Velocity as *AF*, is equal the Magnitude of the direct Stroke with a Velocity as *AC*.

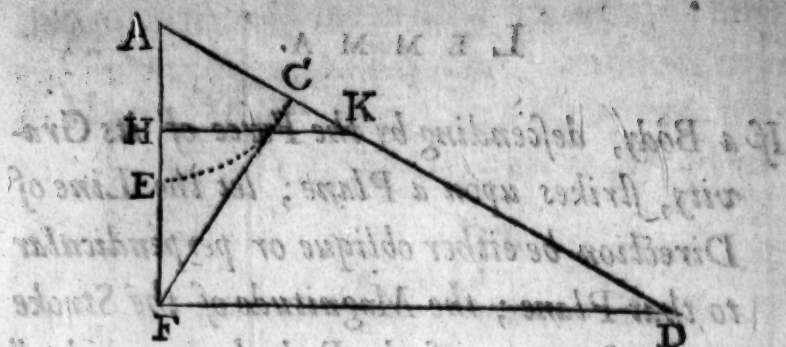
LEMMA.

L E M M A.

If a Body, descending by the Force of its Gravity, strikes upon a Plane; let the Line of Direction be either oblique or perpendicular to that Plane; the Magnitude of the Stroke is the same, as if the Body had moved all along in that same Direction, with a Degree of Velocity, equal to that which is gotten by the accelerated Motion, at the Point where the Line of Direction meets the Plane.

FOR the moving Body and the Line of Direction, being each the same, nothing but a different Velocity in striking upon the Plane can make any Alteration in the Magnitude of the Stroke. But whether the Body moves with an equable or with an accelerated Motion, the Velocity with which it strikes upon the Plane is the same, if the Velocity of the uniform Motion be equal to that gotten by the accelerated, at the Point where the Line of Direction meets the Plane. Therefore, &c. $Q : E : D$.

THEOR.



THEOR. I.

If a Body moves in the Directions AD , AF , from the Point A , by the Force of its Gravity, (FIG. V.) the Magnitude of the Stroke made with the oblique Direction AD , shall be to that made with the Direction AF , perpendicular to the Horizon, as $AF:AD$.

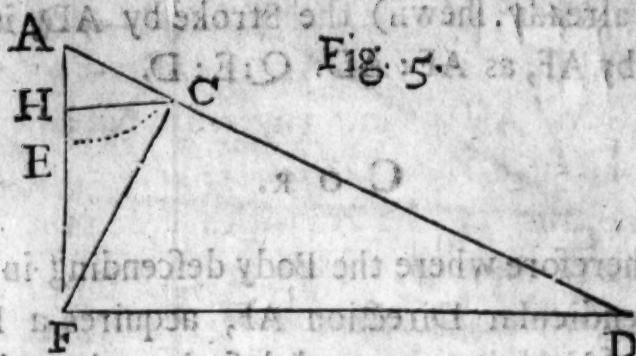
FOR (by the foregoing Lemma) the Strokes made with the oblique and perpendicular Directions, are the same as if the Body had moved uniformly all along, with a Degree of Velocity equal to that gotten by the accelerated Motions, at the Points where the Lines of Direction meet the Plane. But the Velocities gotten (by the Descents) at the Points F and D , where the Lines of Direction meet the Plane, are equal (by what is shewn in Galileus's 3d Mechanick Dialogue. Therefore the Strokes are the same, as if they were made in those

Therefore where the Body descending in the perpendicular Direction AF, acquires a Degree of Velocity, as much less than that which it acquires by the Descent thro' the whole Line AF, (or which is all one, by the Descent thro' AD) as AF is less than AD; I say, where it acquires such a Degree of Velocity; there the direct Stroke made upon a Plane drawn thro' that Point, parallel to FD, shall be equal to the oblique Stroke made in the Direction AD upon the Plane FD; the Body in both Cases moving freely by the Force of its Gravity, as was supposed in the *Theorem*. This is demonstrated in all Respects like the 9th *Caroll*. of the foregoing Problem.

From hence we may easily solve the following Problem.

continued Proportions. The Point H shall be
that required.

P R O B.



PROBLEM.

A Body moving by the Force of its Gravity in the Direction AD (FIG. V.) 'tis required to determine the Point K in the perpendicular Direction AF; so that the Magnitude of the direct Stroke made upon the Plane KH, by the Descent thro' the Line AK, shall equal the Magnitude of the oblique Stroke upon the Plane FD, by the Descent thro' AD.

TAKE $AE=AC$, and cut the Line AF so in the Point H, that AF, AE, AH, may be continual Proportionals. The Point H shall be that required.

Let v = the Velocity acquired by the Descent thro' AH, and V = that acquired by the Descent thro' the oblique Line AD; and W = that

that acquired by the Descent thro' AF. By the Construction $AF : AE :: AE : AH$, therefore $AF : AE :: AF : AH$, and $AF : AE :: \sqrt{AF} : \sqrt{AH}$, that is (since $AE = AC$) $AF : AC :: \sqrt{AF} : \sqrt{AH}$. Now (by what *Galileus* has demonstrated *Theor. 2. Dial. 3.*) $W : v :: \sqrt{AF} : \sqrt{AH}$, that is, the Velocities are in the subduplicate Ratio of the Spaces run, by Equality of Proportion, therefore $W : v :: AF : AC$. But the Velocities gotten by the Descents thro' the Lines AF and AD are equal, by a Proposition of *Galileus's* quoted already) that is $W = V$; therefore $V : v :: AF : AC$; but $AD : AF :: AF : AC$, therefore $V : v :: AD : AF$. That is, the Velocity gotten by the Descent thro' AD, is as much greater than the Velocity gotten by the Descent thro' AH, as AD is bigger than AF. Therefore (by the forgoing *Corollary*) the Magnitude of the direct Stroke made with the Velocity gotten by the Descent thro' AH upon the Plane CH, shall be equal to the Magnitude of the oblique Stroke made with the Velocity gotten by the Descent thro' AD upon the Plane FD. Q; E : D.

Going back by these same Steps, one may thus determine the Point H. That the Point H may be such as is required,

It must be $V : v :: AD : AF$ By the foremen-
 or $V : v :: AF : AC$ tion'd *Corollary*.

But it is $W : v :: \sqrt{AF} : \sqrt{AH}$ $\left\{ \begin{array}{l} \text{From the Law} \\ \text{of Gravity.} \end{array} \right.$
 And V is $= W$ $\left\{ \begin{array}{l} \text{By Galilaeus's} \\ \text{Demonstration.} \end{array} \right.$

Therefore it is also $V : v :: \sqrt{AF} : \sqrt{AH}$
 Therefore it must be $\sqrt{AF} : \sqrt{AH} :: AF : AC$
 That is, it must be $AF : AH :: AF^2 : AC^2$
 or (if $AE = AC$) it must be $AF : AH :: AF^2 : AE^2$
 Therefore putting $AE = AC$, the Line AF must
 be divided so, that AF , AE and AH may be con-
 tinual Proportionals. $Q : E : I$.

T H E O R. II.

*If a Body descending by the Force of its Gra-
 vity (in any Direction, whether perpendi-
 cular or oblique) strikes upon parallel Planes;
 the Magnitudes of the Strokes (whether di-
 rect or oblique) shall be directly proportio-
 nal to the Times of the Body's Descent in
 those Directions.*

C A S E I.

LET the Body descend in the Direction AF
 (*FIG. V.*) perpendicular to the Horizon-
 tal Lines HC , FD . The Strokes made upon the
 Planes HC , FD , after the Descents thro' AH ,
 FD , are as the Velocities with which they are
 made;

made; that is, as the Velocities gotten by those Descents. But the Velocities (from the Nature of uniformly accelerated Motion) are proportional to the Times. Therefore, &c. $Q : E : D$.

C A S E II.

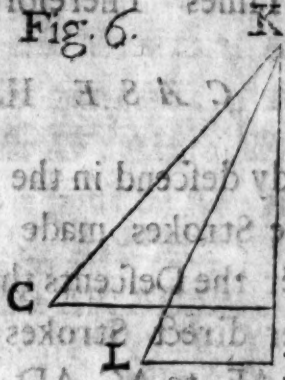
Let the Body descend in the oblique Direction AD. The Strokes made upon the Planes HC, FD, after the Descents thro' AC, AD, are to the former direct Strokes upon the same Planes as AH, AF, to AC, AD, by the foregoing Theorem. Therefore the oblique Strokes are directly proportional to the direct ones, since (from the similar Triangles) $AH : AF :: AC : AD$. Therefore they are also as the Times of the perpendicular Descents, by the former Case. But (by *Theor. III. of Galileus's 3d Mechanick Dial.*) the Times of Descent thro' AC, AD, are to the Times of Descent thro' AH, AF; as the Lines AC, AD, to the Lines AH, AF. Therefore the oblique Strokes upon the Planes HC, FD, are also directly proportional to the Times of the Descent of the Body in the Directions AC, AD. $Q : E : D$.

C O R.

The Strokes either oblique or direct, made upon the the Planes HC, FD, are as $\sqrt{AH}$, and $\sqrt{AF}$, or as $\sqrt{AC}$, and $\sqrt{AD}$; or, which

is all one, they are as the Ordinates (of a Parabola) belonging to the *Abscissæ* AH, AF, respectively.

Fig. 6.



THEOR. III.

A Body descending by the Force of its Gravity in Lines of Direction of equal Length, but unequally enclin'd to Two Planes; the Magnitudes of the oblique Strokes shall be to one another directly as the square Roots of the Cubes of the perpendicular Altitudes of the Lines of Direction.

LET (FIG. VI.) KC=KL, and a Body descending in those Directions, strike upon the Horizontal Planes CD, LE. Draw KDE perpendicular to the Horizon, and consequently so to each of those Planes. Let D express the Magnitude of the direct Stroke made

made upon the Plane LE, O the Magnitude of the oblique one, d the direct Stroke upon the Plane CD, and o the correspondent oblique one.

Then $d : o :: KC : KD$ { By Theor. I.

And $D : O :: KL : KE$

But $d : D :: \sqrt{KC} : \sqrt{KL}$ { By Corollary of Theor. II.

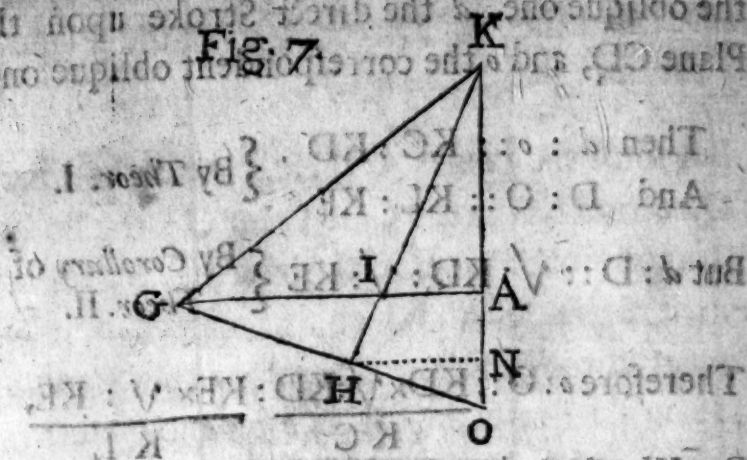
Therefore $o : O :: \frac{KD \times \sqrt{KC}}{KC} : \frac{KE \times \sqrt{KL}}{KL}$

But $KL = KC$, by the Supposition,

Therefore $o : O :: KD^{\frac{3}{2}} : KE^{\frac{3}{2}}$, that is directly as the square Roots of the Cubes of the Altitudes of the Lines of Direction. Q. E. D.

made upon the Plane LE, O the Magnitude of the oblique one, the direct Stroke upon the Plane CD, and the corresponding oblique one.

Fig. 7



THEOR. IV.

A Body descending by the Force of its Gravity in a Line of Direction, which is unequally enclin'd to Two Planes (meeting the same Point) the Magnitudes of the oblique Strokes given upon these Planes, shall be directly as the perpendicular Altitudes of the Line of Direction.

LET (FIG. VII.) the Body descend in the Direction KG, and so strike upon the Planes GA, GH, making the Angle AGH. Let KA be perpendicular to the Horizontal Line GA; and KH be perpendicular to the enclin'd Line GH. The Strokes made by any accelerated Motions are the same as if they were made by equable Motions in the same Directions, and with Velocities equal to those gotten

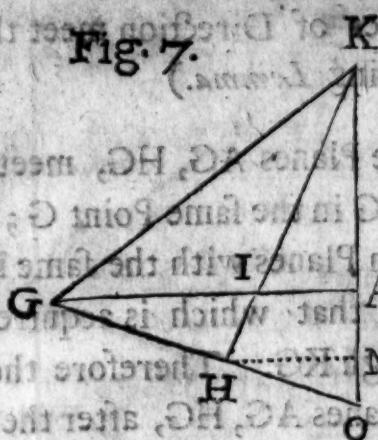
ten by the accelerated Motions, at the Points where the Lines of Direction meet the Planes (by the foregoing *Lemma*.)

But since the Planes AG, HG, meet the Line of Direction KG in the same Point G; the Body falls upon both Planes with the same Degree of Velocity, *viz.* that which is acquired by the Descent through KG. Therefore the Strokes made on the Planes AG, HG, after the Descent, are the same as if they were made in the same Direction, and upon the same Planes, with one and the same Degree of Velocity, in equable Motions. But if they were made with the same Velocities in equable Motions, the Stroke upon AG would be to that upon HG, as KA : KH (by *Cor. IV. Prob. I.*)

Therefore the Strokes are in the same Proportion now; that is, directly as the Altitudes of the Line of Direction above the Two Planes. Q: E; D.

THEOR.

Fig. 7.



THEOR. V.

A Body descending by the Force of its Gravity in Lines of Direction of unequal Length and Inclinations to a Plane, but equally high; the Magnitudes of the oblique Strokes shall be reciprocally as the Lengths of the Lines of Direction.

LET the Body (FIG. VII.) descend in the Directions KG, KI, and let the Angle AGH=IKG; then shall the Angle KGH=KIA.

Now because the Velocity gotten by Descent thro' KI, is equal the Velocity gotten by Descent thro' KG, (by a foremention'd *Theor.* of *Galileus's*) and because the Angle KIA is = KGH; therefore the Stroke with the Direction KI upon GA, is equal the Stroke with the Direction KG upon GH: For the Lines of Direction

rection make the same Angles with the Planes, and the Body falls upon them with the same Degree of Velocity. But (by *Theor. IV.*) the Stroke with the Direction KG upon GA, is to the Stroke with the same Direction KG upon GH, as KA to KH; that is (because of the similar Triangles HKG, AKI) as KI to KG. Therefore also the Stroke with the Direction KG upon GA, is to that with the Direction KI upon GA, as KI to KG; that is reciprocally as the Lengths of the Lines of Direction. Q: E: D

THEOR. VI.

Universally; Any Two oblique Strokes given by a Body, descending by the Force of its Gravity, are in the Ratio compounded of the direct Ratio of the Square Roots of the Cubes of the Altitudes, and the reciprocal Ratio of the Lengths of the Lines of Direction.

This is demonstrated by the Third and Fifth Theorems.

N. B. In like manner we may demonstrate Theorems for the Quantities of Strokes, when the Motions are perform'd according to any other Law of Acceleration; as when the Velocities are in the Duplicate, Triplicate, or any other Multiplicate or Sub-multiplicate Ratio of the

the Times: Concerning which see Dr. Wallis's *Prop. 5. Cap. 10. De Motu Projectorum*. But such Motions as these not being any where found in Nature, we'll pass over that Speculation.

S C H O L. I.

The Proportions relating to the Business of Percussion, already prov'd in the *Corollaries* of *Prob. 1.* may be aptly enough computed by a general *Calculus*, after this Manner. Let *D. d.* express any direct or perpendicular Strokes. *V. v.* the Velocities with which they are made respectively. *O. o.* the correspondent oblique Strokes, made with the same Velocities as the perpendicular ones. *H. h.* the Hypothenufa's or longer Sides that expound the direct Strokes. *P. p.* the shorter Sides or Perpendiculars (from the Lines of the oblique Directions to the Planes upon which the Strokes are made) which expound the oblique Strokes. *M. m.* the Magnitudes or Bulks of the striking Bodies, which we suppose to be Homogeneal. And let it be observ'd (which I mention here once for all) that all the Quantities express'd by the great Letters do belong to one another, and so likewise those express'd by the little ones. Thus *d* expresses the direct Stroke made by the Body whose Magnitude is *m*, its Velocity *v*, the oblique Stroke made by the same *o*, the Line that

that expounds that direct Stroke h , that which expounds the oblique p . And so of the others.

It's obvious, that if $M=m$, then $D:d::V:v$; and if $V=v$, then $D:d::M:m$; so that universally $D:d::M \times V : m \times v$; the Magnitudes of any Two direct Strokes are in the *Ratio* compounded of the direct *Ratio's* of the Magnitudes, and the Velocities of the moving Bodies. Therefore if $D=d$, then $M \times V = m \times v$, and $M:m::v:V$; if the Magnitudes of the Bodies be reciprocally as the Velocities, the direct Strokes shall be equal. Again, $D:O::H:P$, and $d:o::h:p$, (by the general *Theor.* found at *Prob. I.*) but universally $D:d::M \times V : m \times v$, therefore $M \times V : O::H:P$, and $m \times v : o::h:p$, and consequently $O:o::\frac{M \times V \times P}{m \times v \times p}$ universally. That is, the Magnitudes of any oblique Strokes are in the *Ratio* compounded of the direct *Ratio* of the perpendicular Strokes, the direct *Ratio* of the Altitudes, and the reciprocal *Ratio* of the Lengths of the Lines of Direction. Therefore if $M \times V = m \times v$, that is, if $D=d$, then $O:o::\frac{P}{H} \frac{p}{h}::P \times h : p \times H$; that is, the oblique Strokes are in the *Ratio* compounded of the direct *Ratio* of the Altitudes, and the Reciprocal of the Lengths of the Lines of Direction; as was demonstrated at *Cor. VI.*

If

If $P=p$, then $O:o::\frac{1}{H}:\frac{1}{b}::b:H$, which was
 shewn at Cor. V. If $H=b$, then $O:o::P:p$,
 which was prov'd at Cor. III. If $\frac{P}{H}=\frac{p}{b}$, that
 is if $P:p::H:b$, that is, if the Rectangular
 Triangles be similar (and so the Angles of the
 Lines of Direction upon the Planes be equal)
 then $O:o::(M \times V:m \times v)::D:d$, as was de-
 monstrated at Cor. I. and exemplified in some
 particular Instances at Cor. VII. Lastly, To
 find an oblique Stroke equal to a direct one
 made with a less Degree of Velocity, that is,
 the oblique Stroke O equal to the direct Stroke
 d . 'Tis clear, that $\frac{O}{d}=\frac{O}{D} \times \frac{D}{d}$; but $\frac{O}{D}=\frac{P}{H}$,
 and $\frac{D}{d}=\frac{M \times V}{m \times v}$, therefore $\frac{O}{d}=\frac{P \times M \times V}{H \times m \times v}$; conse-
 quently $O:d::P \times M \times V:H \times m \times v$ universally.
 Therefore if it be the same Body, that is, if
 $M=m$, then $O:d::P \times V:H \times v$; consequently,
 if $O=d$, then $P \times V=H \times v$, or $P:H::v:V$.
 That is an oblique Stroke is = to a direct one,
 made with a Velocity as much less than that
 with which the Body moves in the oblique Di-
 rection; as the Perpendicular P is less than the
 Hypothenuse H , which is Cor. IX.

To carry this on to Strokes made by Motions uniformly accelerated: Let us reassume the universal Analogy before laid down, viz. $D:d::M \times V:m \times v$. Where it needs only be noted, that the Quantities V . v . which before express'd the uniform Velocities in equable Motions, do now express the Velocities gotten by the Descents (in accelerated Motions) at the Points where the Lines of Direction meet the Plane. By *Theor. I.* these Analogies hold good in the Case of accelerated Motions too; viz. $D:O::H:P$, and $d:o::b:p$. And therefore (as was shewn in the Case of equable Motions in the foregoing *Calculus*) $M \times V:O::H:P$, and $m \times v:o::b:p$. But (from the known Law of Gravity) 'tis $V:v::P^{\frac{1}{2}}:p^{\frac{1}{2}}$ (the Velocities are in the Subduplicate Ratio of the Lines of Descents) therefore $M \times P^{\frac{1}{2}}:O::H:P$, and $m \times p^{\frac{1}{2}}:o::b:p$. Therefore $O:o::\frac{M \times P^{\frac{1}{2}}}{H}:\frac{m \times p^{\frac{1}{2}}}{b}$ universally. That is, the Magnitudes of any Two oblique Strokes are in the Ratio compounded, of the direct Ratio of the Bodies, the direct Ratio of the $\frac{1}{2}$ Powers of the Altitudes, and the reciprocal Ratio of the Lengths of the Lines of Direction. Therefore if $M=m$, then $O:o::\frac{P^{\frac{1}{2}}}{H}:\frac{p^{\frac{1}{2}}}{b}$; which was the Sixth Theorem foregoing. If $P=p$; then $O:o::\frac{P^{\frac{1}{2}}}{H}:\frac{p^{\frac{1}{2}}}{b}$.

$\frac{1}{H} : \frac{1}{h}$, as was demonstrated at *Theor. V.* If $H = h$, then $O : o :: P^{\frac{1}{2}} : p^{\frac{1}{2}}$, as was shewn at *Theor. III.* If $\frac{P}{H} = \frac{p}{h}$, that is, if $P : p :: H : h$ (that is if the Rectangular Triangles be similar, and consequently the Angles of the Lines of Direction upon the Plane be equal) then it will be $O : o :: P^{\frac{1}{2}} : p^{\frac{1}{2}}$, or $O : o :: H^{\frac{1}{2}} : h^{\frac{1}{2}}$, which was the *Coroll.* of *Theor. II.* Also because $V : v :: P^{\frac{1}{2}} : p^{\frac{1}{2}}$, therefore $O : o :: V : v$ but (putting T , the Times of Descent) 'tis $V : v :: T : t$ (from the Nature of uniformly accelerated Motion) therefore $O : o :: T : t$, which was Case II. of *Theor. II.* and the first Case will be as easily deduced in like manner. Lastly; To find an oblique Stroke equal to a direct one made with a less Degree of Velocity, I proceed altogether as in the *Calculus* for the like Case, in the Hypothesis of uniform Motions, and so come in general Terms to this Conclusion (there investigated) that $O = d$, when $P \times V = H \times v$; that is, when $P : H :: v : V$. And this is the *Corollary* to *Theor. I.* about Motions uniformly accelerated. And why I may by the very same Steps advance towards the Conclusion, in the Case of accelerated Motions, that I did in the Case of equable Motions, I dare presume is evident to any one, that considers but the foregoing *Lemma*, and the *Theorem* that immediately succeeds it.

But

But now to work more particularly and directly to determine whereabouts a heavy Body in its perpendicular Descent shall acquire such a Degree of Velocity, as to make a direct Stroke equal to an oblique one; that is, where O shall

$= d$. It's plain, that $\frac{O}{d} = \frac{O}{D} \times \frac{D}{d}$; but (by *Theor. I.*)

$\frac{O}{D} = \frac{P}{H}$, and $\frac{D}{d} = \frac{V}{v}$ (by what has been shewn

already) and $\frac{V}{v} = \frac{P^{\frac{1}{2}}}{p^{\frac{1}{2}}}$ (by the Law of Gravity)

Therefore $\frac{O}{d} = \frac{P}{H} \times \frac{P^{\frac{1}{2}}}{p^{\frac{1}{2}}} = \frac{P^{\frac{3}{2}}}{H \times p^{\frac{1}{2}}}$ consequently

$O : d :: P^{\frac{3}{2}} : H \times p^{\frac{1}{2}}$. Therefore when $O = d$. then

$P^{\frac{3}{2}} = H \times p^{\frac{1}{2}}$, and so $P : H :: p^{\frac{1}{2}} : P^{\frac{3}{2}}$. Therefore if

(*FIG. V.*) we put $AF = P$, and $AD = H$, then p

($= AH$ the Length in the Perpendicular that is

sought) must be such, that $AF : AD :: \sqrt{AH} :$

$\sqrt{AF}$. But $AF : AD :: AC : AF$, therefore AH

or p must be such, that $AC : AF :: \sqrt{AH} :$

$\sqrt{AF}$. That is (putting $AE = AC$) we must have

$AE^2 : AF^2 :: AH : AF$, or $AE^2 = AF \times AH$, and

so the Lines AF . AE . AH must be continued

Proportionals; which was the Construction be-

fore propos'd. And the Amount of it is no

more than to let fall the Perpendicular CH

from the Point C , which determines the Point

H that was sought; since 'tis certain (from the

similar Triangles) that $AF : AC :: AC : (or$

$AE) AH$. $Q : E : L$.

S C H O L. II.

Having at *Theor.* II. made some Comparison between the Times of a heavy Body's Descent, and the Magnitudes of the Strokes made upon a Plane after those Descents; it may not be amiss to pursue that Matter a little further. All the Symbols standing as in the foregoing *Scholium*; let T, t , express Times of Descent, in the oblique Lines H, h .

1. 'Tis certain, that when $P = p$, then $O : o :: h : H$ { By *Theor.* 5.
But also when $P = p$, then $T : t :: H : h$ { By *Theor.* 3.
 $P : t :: H : h$ { *Galil.* 3d *Dial.*

Therefore in this Case the Quantities of the oblique Strokes are reciprocally as the Times of Descent.

2. If $H = h$, then $O : o :: P^{\frac{3}{2}} : p^{\frac{3}{2}}$ { By *Theor.* 3.
But also when $H = h$, then $T : t :: p^{\frac{1}{2}} : P^{\frac{1}{2}}$ { By *Theor.* 4.
 $p^{\frac{1}{2}} : P^{\frac{1}{2}}$ { *Galil.*

Therefore in this Case the Quantities of the oblique Strokes are reciprocally as the Cubes of the Times of Descent.

3. When neither $H = h$, nor $P = p$, { By *Theor.* 6.
then universally $O : o :: h \times P^{\frac{3}{2}} : H \times p^{\frac{3}{2}}$ {
But 'tis also universally, $T : t :: H \times p^{\frac{1}{2}} : h \times P^{\frac{1}{2}}$ { By *Theor.* 5.
 $H \times p^{\frac{1}{2}} : h \times P^{\frac{1}{2}}$ { *Galil.*

So that the Magnitudes of the *oblique* Strokes are universally in the *Ratio* compounded, of the Reciprocal, and the Triplicate Reciprocal *Ratio* of the Times of Descent; that is, Reciprocally as the Biquadrates of them. And this Rule may be laid down as an universal one to express the

Proportion in Lines, viz. $\frac{O}{o} : \frac{T}{t} :: \frac{h \times P^3}{H \times p^3}$;

$\frac{H \times P^3}{h \times p^3} :: b^2 \times P^2 : H^2 \times p^2$; or $O \times t : o \times T :: b^2 \times P^2 : H^2 \times p^2$;

$\frac{H^2 \times p^2}{h^2 \times P^2}$;

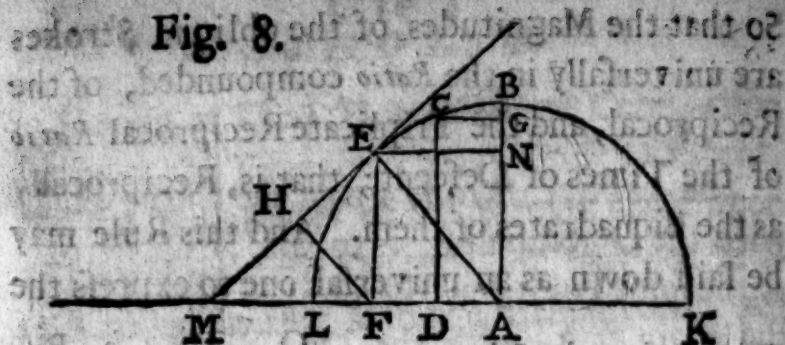
with the *Lines* OH and oh .

The Quantities of Strokes made upon Curves, may be computed as well as those upon right Lines. For the Stroke upon any Point of a Curve is the same with that upon a Tangent at the same Point: Since the Declivity of a Curve and its Tangent are the same at the Point of Contact. *Prop. 15. Cap. 2. of Dr. Wallis's Mechanicks.* Upon this Head therefore I think it not improper to subjoin a *Theorem* or two.

E 2

THEOR.

Fig. 8.



T H E O R. VII.

In a Circle, the oblique Stroke made upon the Curve at any Point as E (FIG. VIII.) with the Direction of the Ordinate to that Point FE; is to a direct Stroke made at the same Point, and with the same Velocity (which I every where suppose) as the Ordinate FE to the Radius EA.

Letting fall FH perpendicular from F to the Tangent ME; 'tis plain by what has been shewn, that the oblique Stroke upon the Plain ME, with the Direction FE, is to the direct one as FH : FE. But the Stroke upon the Tangent, and the Curve is all one at the Point of Contact E; the oblique and direct Strokes therefore upon the Curve are as FH and FE. But the Triangles FHE, AFE are similar; and FH : FE :: FE : EA. Therefore, &c. Q: E: D.

C O R. I.

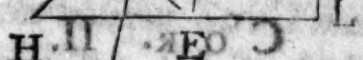
The oblique Stroke upon the Curve at E, with the Direction FE, is to the direct Stroke at the same Point; as the oblique Stroke upon the Plain at A, with the Direction EA, is to the direct Stroke at the same Point A. For both Proportions are that of FE : AE.

C O R. II.

The oblique Strokes made with the same Velocity upon the several Points of the Curve, and with the Directions of the Ordinates; are one to another directly as those Ordinates, or as the Sines of the correspondent Arches. Thus the oblique Stroke at E, is to that at C, as the Sine of the Arch LE, to the Sine of the Arch LC.

THEOREM 3.
Let GE be any Diameter of the Parabola BGC (FIG. IX) whose Axis is BE. Vertex B, Focus D. Conceive a Stroke made in the Direction EG upon the Curve at G; or which is all one, upon the Tangent GA at that Point. Draw DG perpendicular from the Focus to the Tangent; likewise EF perpendicular to the same from the Point E.

Fig. 9.



THEOR. VI

The oblique Strokes made upon the Curve of a Parabola, with the Directions of the Diameters of that Parabola; are ever in the Subduplicate Ratio of the direct Strokes.

L Et GE be any Diameter of the Parabola BGH (*FIG. IX.*) whose Axe is BL, Vertex B, Focus D. Conceive a Stroke made in the Direction EG, upon the Curve at G, or which is all one, upon the Tangent GA at that Point. Draw DC perpendicular from the Focus to the Tangent; likewise EF perpendicular to the same from the Point E.

By

By what has been hitherto shewn, the Magnitude of the oblique Stroke at G is to the Magnitude of the direct one, as $FE : GE$. But from the Nature of the Parabola, the Angle FGE is always $= CGD$, and those at F and D are right ones by the Construction; therefore the Triangles FGE, and CGD are always similar. Therefore since $FE : GE :: CD : DG$, the Magnitude of the oblique Stroke, is to that of the direct always as $CD : DG$. But from the Nature of the Parabola DG is ever as CD^2 , that is the Distance of any Point in the Curve from the Focus, is ever as the Square of the Perpendicular from the Focus to the Tangent at that same Point (which Property our *Celebrated Author* demonstrates at *Leim. 14. Sect. 3. Lib. 1.* and which we shall afterwards give the Investigation of.) Therefore the oblique Strokes are ever in the Subduplicate Ratio of the direct ones. $Q : E : D$.

C O R. I.

'Tis manifest that the same Proportion holds with respect to the oblique Strokes made with the Directions of Lines, drawn out of the Focus to any Points in the Curve.

C O R. II.

If the direct Strokes be made with the same Velocity and so be equal, the oblique ones shall be equal all over the Parabola. For they are still in the $\frac{1}{2}$ Ratio of the direct ones; and to be ever in any certain Ratio of Equals, is to be always equal.

T H E O R. IX.

If the Curve in FIG. IX. were either of the other Conick Sections, one of whose Foci were D, as before, and the other Focus were at the Point E (falling either within or without the Section) the Proportion of the direct and oblique Strokes made upon the Curve wou'd be as before, viz. as the Distance from either Focus to any Point in the Curve is to a perpendicular from the same Focus, let fall upon the Tangent at that Point.

THIS needs no Demonstration. But then, as in the Ellipses, the Perpendicular from the Focus to a Tangent at any Point is still in less, and in the Hyperbola still in more, than the $\frac{1}{2}$ Ratio of the Distance of that Point from the Focus (of which we may have Occasion to speak more hereafter) so in the former Curve will

Line of Direction in B: And Lastly, (to complete the Parallelogram) draw the Lines CD and AD, parallel to AB and BC respectively. The Body now is to be conceiv'd as sustain'd by Three Powers (one of which is equivalent to the other Two) one, the Power, whose Direction is AB; the other, the Force of Gravity, whose Direction is AD; and (in the Room of) the third, the Resistance or Contranitence of the Plane, whose Direction is AC perpendicular to it. Therefore (by the third general *Theor.* after the *Laws of Motion*) the Power at B is to the Force of Gravity, as the Sine of the Angle CAD, to the Sine of the Angle ACD; that is, as CD: DA. Q: E: I.

C O R. I.

If the Line of Direction AB were parallel to the inclin'd Plane HF, then the Angle CAB would be a Right one, for AC is perpendicular to HF. Consequently the Rectangular Triangle CAB, will be similar to the Rectangular Triangle FGH. From thence $AB:BC::$ (that is, $CD:DA::$) $FG:FH$; so that the Power is to the Force of the Weight, as the Sine of the Angle of Elevation (FHG) to the *Radius*.

C O R.

C O R. II.

If the Line of Direction AB were parallel to the Base HG, then the Angle DAB would be a Right one; and consequently $CD : DA :: FG : HG$; and so in this Case the Power is to the Force of the Weight, as the Sine of the Angle FHG, to the Sine of the Angle HFG.

C O R. III.

That Power which sustains a Body upon an enclin'd Plane, having its Line of Direction parallel to the Plane; is less than the Power which sustains the same Body upon the same Plane, with a Direction parallel to the Base. For in the former Case the Proportion is as $FG : FH$, and in the latter as $FG : HG$. And 'tis evident that 'tis as much less, as HG is less than FH.

C O R. IV.

The Force by which the Body endeavours to descend upon the enclin'd Plane, is to the Force by which it endeavours to descend in a Perpendicular, as $CD : DA$ universally (which may be easily determin'd to the particular Cases of Cor. I. and II.) For the Power is to the absolute Force of the Weight (that by which it endeavours to descend in a perpendicular Direction)

rection) as $CD : DA$, by what has been shewn. But the Power sustains, (and therefore is equal to) the Force by which it endeavours to descend upon the inclin'd Plane. Therefore these Forces of the Body are also in the same *Ratio*.

C O R. V.

From the same Principles we may compute the Proportions of the Power to the Gravity of the Weight; when the Inclination of the Plane (upon which the Weight is sustain'd) is continually changing; that is, when we imagine the Body to be held successively upon the several Points of any Curve Surface; in which Case the Tangents to those Points, are the changing inclin'd Planes that we are to consider.

Thus we may conceive the Line HF (*FIG. X.*) to be the Tangent of some Curve (whose Vertex is in some Point between F and G) FG the Subtangent, and HG an Ordinate from the Point of Contact H . 'Tis evident, that to sustain the Weight A continually upon the Curve, there must be a different Power for every different Point where it rests. And the Proportion of the Power to the Gravity of the Weight; for every different Point will be as the Subtangent to the Tangent, in the Case of *Cor. I.* or as the Subtangent to the Ordinate, in the Case of *Cor. II.* Therefore applying the Subtangents and the Tangents, or the Subtan-

gents

gents, and the Ordinates, to some common Axe; the Curvilinear Figures thence arising, will give the Summatory *Ratio* of the Powers to the Force of Gravity, for all the Points of Application thro' the Curve, the *Pondus* rests upon.
Ex. gr.

If FH were the Tangent of a Parabola, whose Parameter $= r$, and Abscisse $= x$. The Subtangent than will be $= 2x$, and the Tangent $= \sqrt{rx + 4xx}$, or taking the Halfs of both, we have x , and $\sqrt{\frac{r}{4}x + xx}$, for the Ordinates of Two Curves. Applying these continually therefore to the Abscisse (x) of the Parabola, we shall have an Ifosceles Triangle, and an equilateral Hyperbola; for 'tis plain that $\sqrt{\frac{r}{4}x + xx}$ is the Expression of the Ordinate of an equilateral Hyperbola, whose Transverse, Conjugate or Parameter are either of them equal to $\frac{r}{4}$. If the Line of Direction were parallel to the Base HG, which is the Ordinate; then the Curve being supposed a Parabola, as before, the Two emerging *Areas*, would be an Ifosceles Triangle, and that very same Parabola, both having the same Abscisse (x .) And in either Supposition the *Areas* thus made express the *Ratio Totalis* of the sustaining Powers to the Force of Gravity.

To

To express these Matters in more general Terms; Let P, p stand for Powers, G, g the absolute Gravities or Forces of any *Pondera* sustain'd upon the enclin'd Planes. L, l the Lengths of those Planes themselves, A, a their perpendicular Altitudes, B, b their Bases.

When the Line of Direction is parallel to the Plane, then (by *Cor. I.*) we have $P : G :: A : L$, and $p : g :: a : l$, and universally $P : p :: \frac{G \times A}{L} :$

$\frac{g \times a}{l}$. So likewise $G : g :: \frac{P \times L}{A} : \frac{p \times l}{a}$. Therefore if $P = p$, then $G : g :: a \times l : A \times L$. If $G = g$, then $P : p :: l \times A : L \times a$. If $L = l$, then $P : p :: G \times A : g \times a$, and $G : g :: \frac{P}{A} : \frac{p}{a}$. If $A = a$, then $P : p :: \frac{G}{L} : \frac{g}{l}$, and $G : g :: P \times L : p \times l$. If $\frac{A}{L} = \frac{a}{l}$, that is, if $L : A :: l : a$ (and so the Rectangular Triangles be similar) then $P : p :: G : g$.

When the Line of Direction is parallel to the Base, then (by *Cor. II.*) we have universally $P : p :: \frac{G \times A}{B} : \frac{g \times a}{b}$; and so likewise $G : g :: \frac{P \times B}{A} : \frac{p \times b}{a}$.

And from hence the like Varieties of Proportions may be deduced as before, and both put into *Theorems* in Words at length, by any one that has a mind to it.

I shall add one or two *Problems* upon this Head of the enclin'd Plane, and so leave it.

P R O B. I.

A Body being sustain'd by a Power up an enclin'd Plane, 'tis required to find what Proportion another Body must bear to it, whose absolute Momentum or Force of descending in a Perpendicular, shall be equal to the Momentum of the other Body upon the enclin'd Plane.

I'LL take the Line of Direction here, parallel to the Plane; and consequently use the Symbols in the foregoing *Schol.* that relate to Cor. I.

Let M express the Magnitude of the Body held upon the enclin'd Plane (at *FIG. X.*) and m the Magnitude of the Body sought; and let their absolute Gravities be G . g . respectively.

'Tis clear from *Corolls.* 1st and 4th, that the Expression of the Force by which the Body endeavours to descend upon the enclin'd Plane, is =

$\frac{G \times A}{L}$. Put this equal to g , the absolute Momentum, or Force of Perpendicular Descent in

the Body sought. Then $\frac{G \times A}{L} = g$, and $G \times A =$

$L \times g$. But $G : g :: M : m$ (the Gravities of Homogeneous

homogeneous Bodies are proportional to their Magnitudes or Bulks) therefore $M \times A = m \times L$, and so $M : m :: L : A$. That is, a Body as much less than the Body proposed, as the Height of the inclin'd Plane is less than the Length of it, will be that requir'd. Q. E. I.

If the *Momentum* of a Body held upon an inclin'd Plane were not to be equal, but in any *Ratio* assign'd, to the absolute *Momentum* of another Body, *Ex. gr.* in the *Ratio* of $p : q$. Then it wou'd be $\frac{G \times A}{L} : g :: p : q$, and $g \times p = \frac{G \times A \times q}{L}$,

that is, $m \times p = \frac{M \times A \times q}{L}$, from whence $M : m ::$

$p : \frac{A \times q}{L} :: L \times p : A \times q$. That is, the Bodies must be in the *Ratio* compounded of the direct *Ratio* of L to A , and the Direct of p to q . *Ex. gr.*

Suppose $p : q :: G : \frac{G \times A}{L}$ (that is, as the absolute *Momentum* of the Body M , to its *Momentum* upon the inclin'd Plane. Then $\frac{G \times A}{L} :$

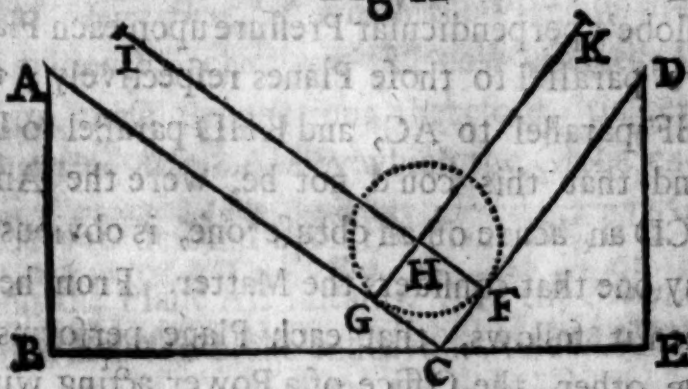
$g :: G : \frac{G \times A}{L}$, that is, $G : \frac{G \times A}{L} :: \frac{G \times A}{L} : g$, in

which Case the *Momentum* of the Body M upon the inclin'd Plane, comes to be a mean Proportional between its own absolute *Momentum* and the absolute *Momentum* of the

Body m . Therefore $G \times g = \frac{G^2 \times A^2}{L^2}$, that is, $g \times L^2 = G \times A^2$, or by equivalent Substitution, $m \times L^2$

$m \times L^2 = M \times A^2$; from whence $M : m :: L^2 : A^2$
That is, if the Body M be to the Body m , as the
Square of the Length of the Plane, to the
Square of the Height; then the *Momentum* of
the Body M upon the enclin'd Plane shall be a
mean Proportional, between its own absolute
Momentum, and the absolute *Momentum* of the
Body m ; or *Vice Versâ*.

Fig. 11.



PROB. II.

A Globe resting upon Two enclin'd Planes,
which cut one another at Right Angles; 'tis
requir'd to find what Proportion of the Pres-
sure, or Burden each Plane sustains.

LET the Globe H rest upon the Two en-
clin'd Planes (FIG. XI.) AC and CD ;
and let the Angle ACD be a Right one. Using
the Symbols as in the foregoing Schol. let G ex-

F

press

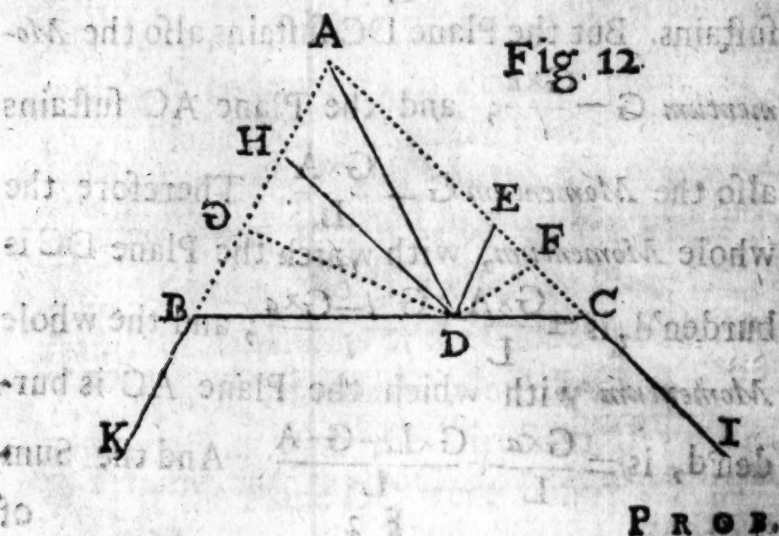
press the absolute Gravity, or total *Momentum* of the Globe, which is shared betwixt the Two Planes (each of which bears its Part) but in what Proportion, we are now to determine. Let $AC=L$, $AB=A$: $DC=l$, $DE=a$, according to the Intent of the *Schol.* The Angle ACD made by the Two Planes, being a Right one by the Hypothesis, 'tis evident, that the Quadrilateral Figure $GHFC$ is a perfect Square; or, which is all one, that the Lines IHF , KGH , which are the Lines of the Direction of the Globe's perpendicular Pressure upon each Plane, are parallel to those Planes respectively; viz. IHF parallel to AC , and KHD parallel to DC . And that this cou'd not be, were the Angle ACD an acute or an obtuse one, is obvious to any one that considers the Matter. From hence then it follows, that each Plane performs to the other, the Office of a Power acting with a Direction parallel to the Plane. Thus the Plane AC sustains all that Force of the Globe, by which it endeavours to descend upon the inclin'd Plane DC , and which Force, if the Plane AC were taken away, or Powers (acting with the Direction KG parallel to the Plane DC) wou'd sustain. So likewise the Plane DC , sustains the *Momentum* of the Globe upon the Plane AC , which *Momentum* wou'd be ballanc'd by a Power acting with the Direction IF parallel to AC , if the Plane DC were remov'd. But be-

sides

sides these Pressures, each Plane is burden'd with one of another kind, and that is the Difference between the total absolute *Momentum* of the Globe and that relative or partial *Momentum* with which the Globe endeavours to descend upon that Plane. For if either of the Planes were taken away, and a Power was substituted in the room of it; then tho' that Power sustain'd the Burden which press'd upon the Plane before, yet the Body still resting upon the other Plane, that Plane is press'd by it; and the Quantities of that *Momentum* by which the Plane is press'd is evidently what was just now asserted. To state these Proportions, therefore, 'tis certain that the *Momentum* by which the Globe endeavours to descend upon the Plane DC, is $= \frac{G \times a}{l}$; this the Plane AC sustains. And the *Momentum* by which the Globe endeavours to descend upon the Plane AC, is, $= \frac{G \times A}{L}$; this the Plane DC sustains. But the Plane DC sustains also the *Momentum* $G - \frac{G \times a}{l}$; and the Plane AC sustains also the *Momentum* $G - \frac{G \times A}{L}$. Therefore the whole *Momentum*, with which the Plane DC is burden'd, is $= \frac{G \times A}{L} + \frac{G \times l - G \times a}{l}$; and the whole *Momentum* with which the Plane AC is burden'd, is $= \frac{G \times a}{l} + \frac{G \times L - G \times A}{L}$. And the Sum of

of these Two Aggregates of *Momenta* (with which both the Planes are burden'd) casting off Contradictories, will be $= \frac{G \times l \times L + G \times l \times L}{L \times l} = 2G$ = to twice the absolute *Momentum* of the Globe; which is absurd and impossible. For the absolute *Momentum* of the Globe is parted between the Two Planes; and what they sustain cannot possibly exceed that. We are therefore (as the excellent Mr. *Leibnitz* shews *Act. Lips.* for *Novemb.* 1685.) to take the Halfs of these Sums of *Momenta*, or (which amounts to the same) an Arithmetical Mean betwixt them.

And then they will be $\frac{G \times A}{2L} + \frac{G \times l - G \times a}{2l}$, and $\frac{G \times a}{2l} + \frac{G \times L - G \times A}{2L}$, the Sum of which is $= G$, the absolute *Momentum* of the Globe. The *Momentum* of the Globe then, with respect to each Plane, is determin'd. $Q : E : F$.



P R O B. III.

*Two Powers sustaining each other upon a Vectis,
'tis requir'd to find the Proportion of one to
the other.*

LET BC be a Vectis, (FIG. XII.) whose Fulciment is D, and let the Powers K and I, acting with the Directions KB, IC, sustain each others Forces, or be in *Equilibria* about the Fulciment D. Produce the Lines of Direction till they meet in A, and draw AD: Likewise draw DE, DH parallel to the Lines of Direction, and DF, DG perpendicular to them. The Fulciment, or fix'd Point D, is here in the stead of a Third Power, and its Line of Direction is DA, drawn from D to A, the Concourse of the Lines of Direction of the other Two Powers I and K. The Vectis therefore being held steady by these Three Powers; Two of them (which draw it with the Directions from A to B, and from A to C) destroy the Effect of the Third, which is to be imagined to draw it with a Direction from D to A. For the Resistance of the Fulciment D (as the Vectis is press'd down upon it by the other Two Powers) is as much as another Power drawing it upwards, with a Direction from D to A, the Concourse of the other Two Directions. Therefore (by Cor. II. of Theor. III.)

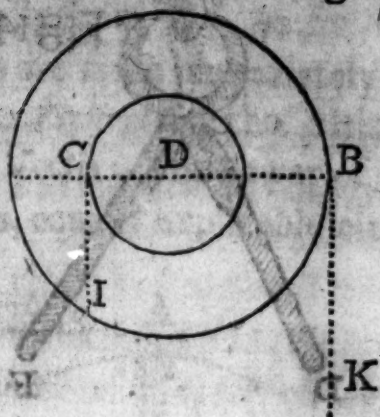
Power I : Power K :: Sine Angle HAD : Sine Angle EAD (or HDA) that is, as HD : ED. But now the Triangles HDG, EDF are similar ; for the Angles at G and F are Right ones by the Construction, and because of the Parallelogram, AED=AHD, and consequently FED=GHD ; therefore the Triangles are similar. Therefore HD : ED :: GD : FD, and consequently, Power I : Power K :: GD : FD ; that is, the Powers are reciprocally as the Distances of their Lines of Direction from the Fulciment. Q. E. I.

C O R. I.

If the Lines of Direction BK and IC were parallel to each other, and perpendicular to the Horizon, then GD and DF would be coincident with DB and DG. Therefore then the Powers would be reciprocally as the Distances (of the Points of Application) from the Fulciment. When the Lines of Direction of the Powers are parallel in the Vectis, then if instead of Two Powers, there were Two Weights appended to the Points B, C, the Machine would become a common *Libra*, the Law of which would be as in the foregoing *Corollary*, viz. The *Pondera* (being in *Equilibrio*) are reciprocally as their Distances from the Center of the *Libra*. Q. E. I.

From

Fig. 13.



From the Explication of the Vectis we may without any further Trouble, deduce the Property of the Wheel, or the Axis in *Peritrochio*. For if DC be the *Radius* of the Axis to which a *Pondus* is applied in C; and DB the *Radius* of the Wheel, or outer Circle it self, to which a Power is applied in B. The Line CB then may be consider'd as a Vectis, of which the Fulciment is D; and the Power ballancing the Weight, that is, the Machine being held perfectly steady and immovable, the Proportion of the Power to the Weight will be always as the Distances of the Lines of Direction from the Fulciment D, reciprocally; that is, as DB, CD reciprocally, or, which is all one, those Distances being the *Radij*, they will be as the Peripheries of the Wheel and the Ax reciprocally. Q: E: I.



From these Principles are also to be deduced the Powers of those Vulgar Machines, Scissars, Pinchers, Sheers, Forcipes, the Crane, the Gimblet, the Hammer (as used and applied to the drawing out of a Nail) with various others. The Four first are manifestly a Pair of Vectes, placed cross-wise, and intersecting each other in their common *Fulcrum*. Thus if *ab CDE* represented a Machine of that kind, where *ba* is the Obex or Body to be cut or compress'd, then *DC* and *EC* being imagin'd to be produced directly to *a* and *b*, we have here Two Vectes in a cross Position, *viz.* *DCa*, and *ECb*, the common *Fulcrum* of which is *C* (the Center of the Rivet) and the moving Powers being applied at *D* and *E*, the other Extremities *a*, *b*, are applied to the Obstacle. From whence the Power of this sort of Machine is easily estimated. The Body and the moving Powers will sustain one another, when the Power is, to the Difficulty or Re-

Resistance of the Obstacle, as the Distance of each reciprocally from C. As for the latter Machines, they are more immediately reducible to the Axis in *Peritrochio*, tho' ultimately resolv'd into the Vectis: The Power of the Hammer is, perhaps, equally explicable either Way.

Fig. 15.

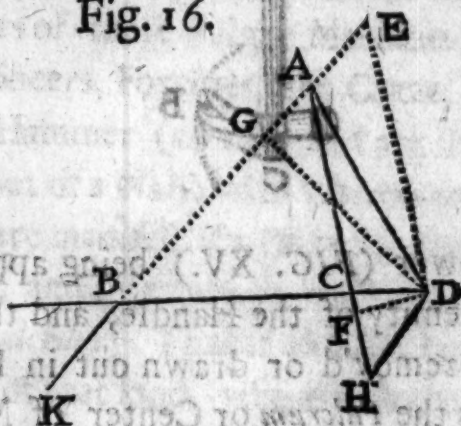


Thus the Power (*FIG. XV.*) being applied in A, the Extremity of the Handle, and the Obstacle to be remov'd or drawn out in B, then the End C is the *Fulcrum* or Center of Motion. And the Power moving in a Circumference, whose *Radius* is CA and the Obstacle (when mov'd) in another, whose *Radius* is CB. Hence if the Forces (of the Power, and of the Resistance of the Obstacle) are reciprocally, as those Peripheries, or their respective *Radij* (which are the Distances from the Fulcriment) they will sustain one another, and be in *Equilibrio*.

The

The Vectis of the first sort has been hitherto explain'd; and that partly in it self, and partly the Machines, that are nearly related to it. For such are all those that have been mention'd under this Head; unless any one thinks it more proper to reduce the Hammer (applied to the Uses just now mention'd) to the Principle of the recurv'd Vectis. However, be it which way it will, the Explication of the other kinds of Vectes will not be improper nor unuseful.

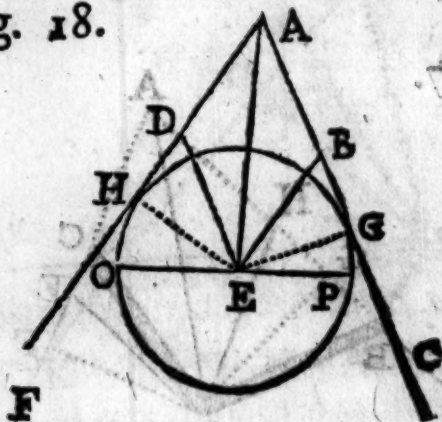
Fig. 16.



Let (*FIG. XVI.*) D be the Fulcrum of the Vectis BCD, and the Powers at K and H, whose Lines of Direction are CB and HC, be conceiv'd to sustain one another upon it. Produce the Line KB till it cuts HC in the Point A, and draw DE parallel to CA, till it meets the Line KBA produc'd in the Point E. Compleat the Parallelogram AEDH, and draw the Diagonal AD. Lastly, From the Point D, let fall DG, DF perpendicular to the Lines of Directions KBA

cing the Lines of Direction till they intersect in A, and then compleating the Parallelogram HADE, whose Diagonal is AD: And, Lastly, Letting fall the Perpendiculars DG, DF, upon the Lines of Direction, 'twill be prov'd, that the Powers are as those Perpendiculars reciprocally. Consequently when the Lines of Direction are parallel, the Powers will be reciprocally, as the Distances of the Points of Application from the Fulciment D, taken in a Line parallel to the Horizon.

Fig. 18.



P R O B. IV.

Two Powers acting with given Directions, sustaining one another upon a Trochlea; it is requir'd to find the Proportion of those Powers.

Suppose the Trochlea (FIG. XVIII.) firmly fix'd by its Center E to some Body that supports it, without any Motion. And let the Powers
at

at F and C, whose Directions are FD, CB, sustain one another upon it. Produce the Lines of Direction till they meet in A, and compleat the Parallelogram ADEB, and let fall the Perpendiculars EG, EH to the Lines of Direction, which being Tangents at H and G, these Perpendiculars will be *Radij*, from the Nature of the Circle. Therefore now (by *Cor. II. of Theor. III.*) Power at C : to Power at F :: as Sine Angle AEB : to Sine Angle BAE; that is, as AB : EB, that is, as ED : EB. But the Triangles EGB, EHD are similar (for the Angles at G and H are Right ones, and EBG = EDH, because of the Parallelogram) therefore ED : EB :: HE : GE; and consequently Power C = Power F. Q : E : I.

C O R.

If the Lines of Direction were parallel, the Law would be the same, as also in this latter Case, if a Weight were substituted in the room of either of the Powers.

S C H O L.

Because the Tangent AH, AG, are equal from the Nature of the Circle, therefore Angle HAE = GAE; and so 'tis presently to be concluded that Power C is = Power F. But to use the perpendiculars from the Center to the Lines of Direction

rection is not only equally clear and demonstra-
 tive, but also a Method vastly more general,
 as being applicable to other Machines where a
 Circle is of no Consideration, and to the Case of
 parallel as well as converging Lines of Directi-
 on. Instances of this were given in the Vectis
 Libra, and Axis in *Peritrochio*. The Trochlea
 it self is no other than a Vectis. And the Case
 just now consider'd, is no other than that of a
 Vectis of the first kind, in which the Fulci-
 ment is equally distant from each Point of
 Suspension.

(for the Angles at G and H are Right Angles)
 $EBG = EDH$, because of the Parallelism
 therefore $ED : EB :: HE : GE$; and conse-
 quently Power C = Power F. $O : E :: I$.

Fig. 19.



If the Lines of Direction were parallel, the
 Law would be the same, as also in this latter
 Case, if a Weight were substituted in the room
 of either of the Powers.

Because the Tangent AG, AG are equal from
 the Nature of the Circle, therefore Angle HAF =
 GAF; and to this presently to be concluded that
 Power C is = Power F. But to use the per-
 pendiculars from the Center to the Lines of Di-
 rection

PROB.

P R O B. V.

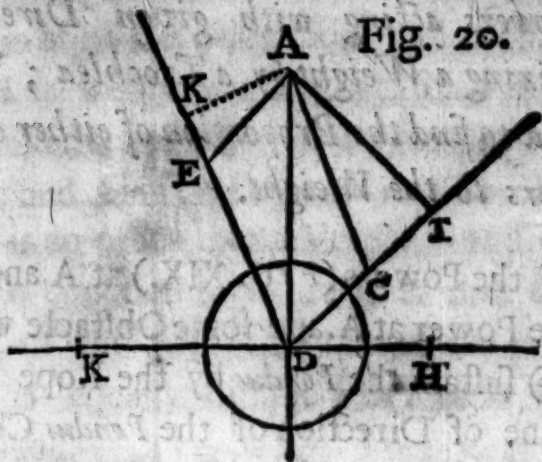
Two Powers acting with given Directions, sustaining a Weight by a Trochlea ; 'tis requir'd to find the Proportion of either of those Powers to the Weight.

LET the Powers (FIG. XIX.) at A and B (or the Power at A, and some Obstacle well fix'd at B) sustain the *Pondus* by the Rope AEDB. The Line of Direction of the *Pondus* CF passes thro' C, the Center of the Trochlea ; and those of the Powers, which are AE and DB being produced, concur in F. Then by *Cor. II. Theor. III.* Power A : to the expended Weight A :: Sine Angle CFD : Sine Angle EFD. But AE and BD being Tangents, 'twill easily be prov'd, that the Angles EFC and DFC are equal. So that the Power is to the Weight, as the Sine of the Angle made by the Lines of Direction to the Sine of that Angle it self. Q. E. I.

When the Lines of Direction EF, DF, are parallel, then EC and CD, the Perpendiculars to them, will lie in one Right Line. And this will be reducible to a Vectis of the second Sort, where one Power is applied between the Fulcrum and the other Power. Consequently the

Power

Power at A will be to the Weight, as the Radius to the Diameter; that is, as 1 : 2.



P R O B. VI.

Two Powers acting with given Directions, sustaining a Weight (without a Trochlea) 'tis requir'd to find what Proportion they bear to one another.

LET the Powers (*FIG. XX.*) at I and K, whose Directions are KD, and ID, sustain the *Pondus* D. Produce the Lines of Direction till they meet in the Center D. Compleat the Parallelogram AEDC. Then by *Cor. II. Theor. III.* the Power at K, is to the Power at I :: as Sine Angle CAD : Sine EDA :: AC : CD, or AE. or which is all one, they are as AI : AK; the Lines AK, and AI being Perpendiculars from A to the Lines of Direction.

C O R.

From hence one may easily compute the Powers of these Rhombs, which the fam'd *Borelli* considers, in order to the Estimation of the Forces of the Muscles; and that, whether single or combin'd together in Chains, according to all the Varieties mention'd in several Propositions in his excellent Book, *De Motu Animalium*. For the Proportion of the Powers not only to one another, but also to the Gravity of the Weight, is easily deduced from that Third General Theorem that has been so often quoted already upon these Occasions.

As for the Powers of the Wedge and the Screw, tho' they are not so immediately and directly explain'd, from the general Principle of the Composition of Forces, as those others that we have consider'd; yet they are included in it, being to be reduced to some one Head or other that is immediately demonstrated from thence. Thus some People have refer'd both these Faculties to that of the inclin'd Plane; while others (whatever they do with the Screw) have reduced the Wedge to a double Vexis. But I am satisfied I can't do better in this Matter, than to give Mr. *Newton's* own Account of them, and refer the Explication of the Wedge to that of the Body pressing upon Two inclin'd

Planes making an Angle with one another, and make the Screw no other than a Wedge impell'd by a Vectis. The celebrated Dr. *Wallis* deduces the Properties of these (as of the rest) from his general *Prop.* which is *Prop. 5. Chap. 2.* of his Tract, *De Motu*. But from which soever of these Principles a Man wou'd explain the Powers of these Machines, he will come to the same Conclusions; viz. For the Wedge; that when the impelling Force is to the Resistance of the Obstacle, as the Thickness of the Wedge, to the Height of the same: And for the Screw; that when the Force that turns the Screw is to the Resistance of the Obstacle, as the Interval of the Two immediately succeeding Turnings of the Spiral, to the Circumference described by the Power in one Conversion of the Screw: I say, that when it is so, the Powers (in these Machines) will be equivalent to the Resistances of the Obstacles, or (if one may so express it) there will be an *Equilibrium* of Forces and Resistances. Or, as our Great Authour expresses it, when the Force by which the Wedge urges the two Sides of the cleft Bodies is to the Force of the Mallet upon the Wedge, as the Progress of the Wedge according to the Determination of the Force impress'd upon it, to the Velocity with which the Parts of the Body give way to the Wedge in Lines perpendicular to the Sides of the Wedge; or when the Force of the Screw

to press the Body, is to the Force of the Hand that turns it round, as the circular Velocity of the Handle, to the progressive Velocity of the Screw towards the compress'd Body; then there will be an *Equilibrium*, or the contrary Forces and Resistances will sustain one another.

But this holds true in all Machines whatsoever, that any Agent and Patient, Force and Resistance, or contrary Force and Force, are then equivalent to, and will sustain one another, when they are reciprocally as their respective Velocities, estimated according to the proper Directions and Determinations of those Forces. For the Action of any Agent being estimated by its Force and Velocity conjunctly, and the Re-action of the Patient or resisting Body, estimated also by the Velocities and the Forces of Resistance in the several Parts (arising from their mutual Attrition, Cohælion, Weight or Acceleration) then Action and Re-action shall be equal to one another throughout all the Machines imaginable; or, in other Words, there will be an *Equilibrium*. The whole Business of *Mechanicks* may therefore be brought into a very narrow Compass, and the Powers of all Instruments whatsoever are included in this one general Analogy that follows. Let *p. r.* express any Powers, moving Forces, or Forces and Resistances; let the Velocity of *p.* be express'd

by m , and that of r by n . Then if $p:r::n:m$, the contrary Forces will sustain one another, because (upon this Supposition) $p \times m \pm r \times n$.

Therefore if $p \times m > r \times n$; that is, if $\frac{r}{p} > \frac{m}{n}$; then the Force p shall overcome the Force or Resistance r . Hence in any Machine the Problem may be solved, *To move a given Weight (or to overcome a given Resistance) with a given Force.*

And this one Application alone were sufficient to shew, how true and certain, how general and extensive that Third Law of Motion is, which tells us, that Re-action is ever equal to Action, &c. But this is but one Instance of the Use of it neither; there are several others besides of no less Consequence and Importance than this, which the Great Author himself has consider'd, and his, and several other Examples relating to those Matters, have been very well and largely explain'd by Mr. Keil; upon which Account I shall say the less to them. The same ingenious Person last mention'd has also in a late Book (*his Introduction to the true Physicks*) made use of the Second Law of Motion as a Principle to establish some important Theorems upon, relating to Accelerated Motion; and the same Law will be of as great Use in demonstrating all the common Theorems about equable Motions. But we are now to proceed with the Corollaries of those Laws.

Second

**Second COROLLARY to the Laws
of Motion.**

The Quantity of Motion, which arises by taking the Sum of the Motion towards the same Part, or the Difference of those made towards contrary Parts, is not changed by the Action of Bodies one upon another.

THAT is, if Two Bodies (moving either towards the same or contrary Parts) strike each other, the Sum of the Motions made towards the same Part, and the Difference of those made towards contrary Parts, will be the same after the Stroke that they were before. Let B & b be Two Bodies, V = the Velocity of B , and v = the Velocity of b , S = the Sum of the Motions (made towards the same Part) D = the Difference of the Motions made towards contrary Parts. 'Tis plain, before these Bodies strike one another, that the Sum of the Motions towards the same Part = S , is = $B \times V + b \times v$, (from that common Principle that the *Momentum* or Quantity of Motion of any Body is = to the Parallelogram under the Quantity of Matter it contains and its Velocity) and the Difference of the Motions towards contrary Parts = D is = $B \times V - b \times v$. Suppose now the Bo-

dies B and b to strike one another, and let the Action of B upon b be put $=m$, the Re-action of b upon B $=n$, then after the Collision the Motion of B $=B \times V \pm n$, and the Motion of $b = b \times v \pm m$, and consequently the Sum or Difference ($=S$ or D) is $=B \times V \pm n \pm b \times v \pm m$, in which Expression the Quantities m and n must be affected with different Signs; that is, if m be positive, n will be negative, and if n be positive, m will be negative, because the Directions of the Action and Re-action are contrary. But (by the Third Law of Motion) Action is ever equal to Re-action; that is, the Quantity $m = -n$, or $n = -m$. Consequently, substituting $-n$ in the room of m , and $-m$ in the room of n , they will destroy one another, and there will remain $B \times V \pm b \times v = S$, and $M \times V - m \times v = D$, the same Values as before. So that the Sum or Difference of the Motions is the same after the Collision that it was before.

From hence the Laws of the Collision and Motion of Bodies may be easily and universally computed.

First, In perfectly hard Bodies; in order to which it's necessary that we premise this

LEMMA.

L E M M A

If a hard Body strikes directly upon another, whether that upon which the Stroke is made be quiescent, or moves more slowly towards the same Quarter, or, Lastly, towards the contrary Part (with a less Motion) after the Stroke they shall both move with the same Degree of Velocity.

LET the Body B strike directly upon *b*. Then (by the Second Law of Motion) the Body *b* shall move according to the Direction of the Force impress'd upon it by the Body B; the Action and Influence of all circumjacent Bodies being supposed to be remov'd. But *b* cannot move slower than B, because of B following it; nor can it move faster, since (by the Hypothesis) we remove all other Causes of its Motion, besides the impelling Body B. Therefore the Bodies B and *b* shall move both with the same Velocity after their Collision.

From hence therefore to find true the Value of that common Velocity, with which the Body shall move after the Stroke: Let this common Velocity be put $=z$. Then $S=B \times V - b \times v$, and $D=B \times V - b \times v$, before the Stroke; and also (by Cor. II. to the Laws of Motion) the Values of S and D continue the same before and after the

Stroke. But after the Stroke $S = B \times z + b \times z$, and $D = B \times z - b \times z$. Therefore $\frac{B \times V + b \times v}{B + b} = z$, the common Velocity for the Sum of the Motions made towards the same Part; and $\frac{B \times V - b \times v}{B - b} = z$, the common Velocity for the Difference of the Motions made towards contrary Parts.

Hence if the Body b were quiescent, then $v = 0$, and so $z = \frac{B \times V}{B + b}$, that is, $V : z :: B + b : B$; the Velocity before the Stroke is to that after it, as the Sum of the Two Bodies to the moving Body. Therefore if $B = b$, then $V : z :: 2 : 1$; that is, if the Bodies are equal, the Velocity after the Stroke will be half the Velocity of the Body, that was supposed in Motion, before the Stroke.

If v and V be both of some real Value, and also if $B = b$, then (for the Case of the Sum of Motions towards the same Part) we have $\frac{BV + Bv}{2B} = z$, or $z = \frac{V + v}{2}$; that is, the Velocity after the Stroke is $\frac{1}{2}$ the Sum of the Velocities before the Stroke. For the Case of the Difference of Motions towards contrary Parts, we have $\frac{BV - Bv}{2B} = z$, or $z = \frac{V - v}{2}$, in which Case the Velocity after the Stroke is $\frac{1}{2}$ the Difference of the Velocities before it.

But

But universally, let the the Bodies be in any Ratio whatsoever, the Value of z will be determined. Suppose $B : b :: p : q$. then $b = \frac{B \times q}{p}$,

which Value substituted in the Equation above, will give the Value of z in this general Supposition. For then we shall find $z =$

$$\frac{pBV + qBv}{pB + qB} \text{ (for the Case of the Sum of Moti-}$$

$$\text{ons towards the same Part) and } z = \frac{pBV - qBv}{pB + qB}$$

(for the Case of the Differences of Motions towards contrary Parts.) And if we suppose b quiescent (as before) and consequently $v = 0$,

$$\text{then } z = \frac{pBV}{pB + qB}, \text{ and } V : z :: p + q : p.$$

In the Case of the Difference of Motions towards contrary Parts, if we put $z = 0$; that is,

if the Bodies, after their mutual Collision, have no Velocity at all, or the contrary Moti-

$$\text{ons have destroy'd one another, then } \frac{pBV - qBv}{pB + qB}$$

$$= 0, \text{ and } p \times V = q \times v, \text{ or (because of } p : q :: B : b)$$

$$B \times V = b \times v; \text{ that is, } B : b :: v : V. \text{ So that the}$$

Bodies being reciprocally as their Velocities (which makes the contrary *Momenta* equal) after the Stroke they will lose all their Motion.

And having thus determin'd what the Velocity must be after the Collision of any hard Bodies, there will be no Difficulty in finding the

Mo-

Momenta, or Quantities of Motion in each of the striking Bodies, after they have thus struck one another; for there is no more to be done, but to multiply the common Velocity into the Magnitude of each Body.

And from hence we may easily determine the Magnitudes of the Strokes made by the Collision of one hard Body with another, and so arrive at those *Theorems* which the celebrated Dr. *Wallis* has given us relating to this Matter, in the *Chap.* of Percussion, which is the 11th of his admirable Tract, *De Motu*. For (according to the Accounts of that excellent Person) the Magnitude of any Stroke is equivalent to twice the *Momentum*, lost from the stronger of the Two Bodies that make the Shock (that is, supposing one to have more Force than the other.) But by the Rules above deliver'd we can find universally what the *Momentum* of any hard Body is after it has struck another, and the *Momentum* before the Stroke being known likewise; 'tis plain, that we can find the Difference between the latter and the former of these *Momenta*; and twice that Difference is determin'd (*Prop.* 5. of the foremention'd *Chap.*) to be equivalent to the Magnitude of the Stroke. For the Doctor brings into the Account of the Stroke, not only what the striking Body loses but also what the Body that is struck receives: Both these (he says) are the Effects of the Stroke,
and

and consequently the Magnitude of it is to be estimated from both together.

Here therefore I will from this Principle deduce Two of his most general Theorems, from whence the Reader may very easily draw the rest himself.

Using the Symbols as before; let the Bodies B. b. whether equal or unequal, move with any Velocities towards the same Parts; and let the Body b. go before, and the Body B. come after with a Velocity V. greater than v. the Velocity of the former. The common Velocity after the Stroke (*viz.* z) = $\frac{BV + bv}{B + b}$; therefore the

Momentum of B after the Stroke, is = $\frac{BBV + Bbv}{B + b}$;

but its Momentum before the Stroke was = BV, therefore the Body B has lost the Momentum

$BV - \frac{BBV + Bbv}{B + b} = \frac{BbV - Bbv}{B + b}$. But the Body b

has acquired just as much; for its Momentum before the Stroke was = bv, and afterwards 'tis = $\frac{BbV + b bv}{B + b}$; therefore it has gotten the Momentum

$\frac{BbV + b bv}{B + b} - bv = \frac{BbV - Bbv}{B + b}$. Consequently,

$\frac{2 BbV - 2 Bbv}{B + b}$ is = the Magnitude of the Stroke.

But 'tis evident, that $\frac{2 BbV - 2 Bbv}{B + b} : BV - Bv ::$

2 b : B + b; that is, the Magnitude of the Stroke, is to the Momentum of the following Body (B)

car-

carried with the Difference of the Velocities, as twice the antecedent Body (b) to the Aggregate of both the Bodies.

In like manner, if the Bodies mov'd towards contrary Parts, it will be found, that $\frac{2BbV + 2Bbv}{B + b} :$

$BV + Bv :: 2b : B + b$; that is, the Magnitude of the Stroke, wou'd be to the *Momentum* of either of the Bodies (*Ex. gr.* B) carried with the Sum of the Velocities, as twice the other Body (*viz.* b) to the Aggregate of both the Bodies.

And from these Two Theorems one may deduce any of the rest, according to the Nature of the Supposition made.

C O R. I.

Joining the Result of both these Analogies together, it follows, that $\frac{2BbV + 2Bbv}{B + b} :$

$\frac{2BbV + 2Bbv}{B + b} :: BV - Bv : BV + Bv :: V - v : V + v$; that is, the Magnitude of the Stroke in the Case of Motions made towards the same Parts, is to the Magnitude of the Stroke in the Case of Motions towards contrary Parts, as the Difference of the Velocities in the former Supposition, to the Sum of the Velocity in the latter; that is, as the relative Velocities of the Bodies in both Cases after the Collision.

C O R.

C O R. II.

In either Case consider'd apart by it self, 'tis plain, that whatever the Velocities are in themselves, yet if the Difference, or the Sum of them continues still the same, the Magnitude of the Stroke shall still be the same also.

And therefore it follows, that the Magnitude of the Stroke is ever proportional to the Difference or the Sum of the Velocities; that is, to the relative Velocity of the Bodies; by which they come towards one another.

From the *Laws of Motion* in perfectly hard Bodies, we come to consider.

2. In perfectly Elastick Bodies; and here 'tis necessary to premise this

L E M M A.

That if Two Bodies perfectly Elastick strike one another, their relative Velocity continues the same after the Stroke, that it was before.

FOR the Nature of a Body perfectly Elastick is to restore it self with a Force equal to that by which it was compress'd. But (in the Hypothesis of the present *Lemma*) the Compressive Force is no other than the Force of the Stroke,

Stroke, by which one of the Elastick Bodies shocks the other.

Now the Magnitude of the Stroke (by the Second *Cor.* foregoing) arises from the relative Velocity of the Bodies, and is proportional to it. Consequently the compressive Force is proportional to that Velocity. But the restitutive Force being equal to the compressive, acting upon the same Bodies, shall produce a relative Velocity that bears the same Proportion to it, that the relative Velocity before the Stroke bore to the compressive Force; therefore it shall produce a relative Velocity equal to the former. And therefore the Body's after Collision shall depart from each other with the same Velocity that they came towards one another with before. $Q: E: D.$

From hence now to determine the Velocities of Elastick Bodies after the Stroke.

All the other Symbols standing as before, let the Velocity of the Body B after the Collision be put $=z$. When the Bodies B. *b.* move towards the same Parts, then their relative Velocity is $=V-v$; and when towards contrary Parts, 'tis $=V+v$. Therefore (by the foregoing *Lemma*) in the former Case, the Velocity of the Body *b.* after the Stroke shall $=z-v-V$; and in the latter Case it shall $=V+v+z$. But (by *Cor. II.* to the *Laws of Motion*) the Sums of the Motions towards the same Parts, or the Differences

ferences towards contrary Parts are equal, both before and after the Stroke. Therefore $BV + bv = Bz + bz + bV - bv$, in one Case, and $BV - bv = Bz + bV + bv + bz$, in the other.

Therefore $z = \frac{BV + 2bv - bV}{B + b}$, in one Case, and $z = \frac{BV - 2bv - bV}{B + b}$, in the other. And put-

ting the Velocity of the Body b after the Stroke $= y$; we have $y (= z + V - v) = \frac{2BV + bv - Bv}{B + b}$,

in one Case; and $y (= z + V + v) = \frac{2BV + Bv - bv}{B + b}$,

in the other Case. So that we have these 2 general Analogies, viz. $z : y :: BV + 2bv - bV : 2BV + bv - Bv$, when the Bodies B . and b . move towards the same Parts; and $z : y :: BV - 2bv - bV : 2BV + Bv - bv$, when the Motions are made towards contrary Parts. And these Values of z and y do give us Dr. Wallis's 9th and 10th Props. Chap. 13. *Mechan.*

And from these general Expressions thus found, all the particular Laws are easily determin'd.

Ex. gr. Suppose the Body b were quiescent; then $v = 0$, and consequently $z = \frac{BV - bV}{B + b}$, and

$\frac{BV - bV}{B + b} : V :: B - b : B + b$; that is, the Velocity of the moving Body B , after the Stroke, is to the Velocity it had before, as the Difference

of

of the Two Bodies to the Sum of them. Again,
 (using the same Supposition) $y = \frac{2BV}{B+b}$, and $\frac{2BV}{B+b}$:
 $V :: 2 B : B+b$; that is, the Velocity of the
 quiescent Body b after the Stroke is to the Ve-
 locity of the striking Body, B before the Stroke,
 as twice the striking Body, to the Sum of both
 Bodies. If $B=b$, then b , supposed quiescent as
 before, z will $= 0$; that is, the Body B after
 the Stroke will lose all its Motion. And all this
 Dr. Wallis shews *Prop. 8. Chap. 13.* concerning
 Elastick Force and Reflection.

Suppose $B=b$, and that the Bodies mov'd to-
 wards contrary Parts. Then $z = -\frac{2bv}{B+b}$, and
 $y = \frac{+2BV}{B+b}$; but because $B=b$, therefore $\frac{-2bv}{B+b}$
 $= \frac{-2Bv}{B+b}$. So that after the Stroke, the Bo-
 dies shall mutually interchange their Velocities,
 the Body B moving with the Velocity of b be-
 fore the Stroke, and the Body b with the Velo-
 city of B before the Stroke. And that they shall
 move towards contrary Parts is manifestly e-
 nough signified by the Difference of the Signs,
 for it is $+V$, and $-v$. And this is *Prop. 7.* of
 the foremention'd *Chap.*

Suppose again $B=b$, and that they mov'd
 towards the same Parts, the Body b going be-
 fore with a Velocity $= v$, and the Body B co-
 ming

ming after with a greater Velocity, viz. V .

Then $z = \frac{+2bv}{B+b}$, and $y = \frac{+2BV}{B+b}$. Therefore

after the Stroke, the Bodies shall move with the Velocities they had before the Stroke, alternately taken, as it was in the Case foregoing, only with this Difference, that there they mov'd towards contrary Parts, and here towards the same Parts, as is plain from the Identity of the of the Signs. And this is *Prop. 6.*

If $B=b$, but the Body b be suppos'd quiescent; then $z=0$, (for in this Case $v=0$, and $bv=0$, and because $b=B$, then $BV-bV=0$, and so the whole

Value of $z=0$) but $y = \frac{2BV}{B+b} = \frac{2BV}{2B} = V$. So that

after the Stroke the Body B at first in Motion shall quiesce, and the Body b at first quiescent shall move, with a Velocity $= V$, the Velocity of B before the Stroke. And this is *Prop. 5.*

If B were $=b$, and also $V=v$, and the Bodies mov'd towards contrary Parts, then 'tis plain, that $z=-v$, and $y=-v$; that is, the Velocity of each Body after the Stroke is the very same it had before, or they are reflected each with those Velocities with which they meet together. Here also the Difference of the Signs discovers, that the reflected Motions are made towards contrary Parts; and the Nature of an Elastick Body (which ever restores it self in the same Direction according to which it was comprest)

as plainly shews, that the Bodies will move in the very same Right Lines after the Stroke, that they did before.

Again, if neither $B=b$, nor $V=v$, yet if the the Bodies be reciprocally as their Velocities; that is, if $B:b::v:V$; in this Case also after the Stroke they shall move towards contrary Parts with the same Velocities they had before. For since (by the Supposition) $B \times V = b \times v$, therefore (the Motions being made towards contrary Parts) z shall $= \frac{-bv - bV}{B + b}$, and y shall $= \frac{+BV + Bv}{B + b}$. But also $\frac{-bv - bV}{B + b} = -V$ and $\frac{+BV + Bv}{B + b} = +v$; therefore the Bodies shall move with the same Velocities respectively towards contrary Parts after their Collision. And this is *Prop. 3. of the foremention'd Chap.*

Having now consider'd these particular Laws of Motion, I proceed to

Third

Third COROLLARY to the general
Laws of Motion or Nature.

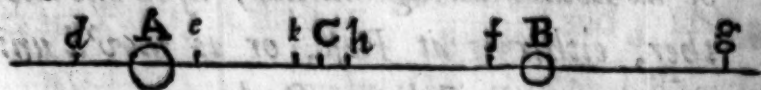
The common Center of Gravity alters not its State of Motion or Rest by the Actions of Bodies upon one another; and therefore (setting aside all external Actions and Impediments) the common Center of Gravity of all Bodies acting mutually upon one another, either is at Rest, or is mov'd uniformly in a Rectilineal Path.

THE common Center of Gravity of Two Bodies, is the Center or fix'd Point of an imaginary *Libra*, about which the Bodies are in *Equilibrio*. This *Libra* is a Line conceiv'd to pass thro' the Center of Gravity of each Body or to be continued from one Body's Center of Gravity to that of the other. Then the Fulciment or fix'd Point (found by taking the Distances of the Centers of Gravity of the Bodies from that Point, reciprocally proportional to the Magnitudes of the Bodies themselves) is what we call the common Center of Gravity of the Two Bodies. So likewise we may make an Estimate of the common Center of Gravity of any Number or System of Bodies, according to this Principle.

The Demonstration of this *Coroll.* will be clear, upon the making out of these Two Proportions.

1. That Bodies moving uniformly, either towards the same, or towards contrary Parts; before they meet and strike one another, their common Center of Gravity will either quiesce, or move uniformly in a Right Line.

Fig. 21.



Let the Bodies (*FIG. XXI.*) be *A. B.* their common Center of Gravity *C.* the Distances of their Center of Gravity from that Point, *AC* and *CB* respectively. And let the small Letters denote different Places of the same Bodies, and of their common Center of Gravity.

From the Nature of the common Center of Gravity, 'tis plain that $A : B :: BC : AC$, from whence $A \times AC = B \times BC$. Now if the Bodies mov'd into the Places *d. g.* or *e. f.* and it were so that $A : B :: gC : dC$, or $A : B :: fC : eC$; then the Motions with respect to the same Point *C*, wou'd still be equal, because $A \times dC = B \times gC$, or $A \times eC = B \times fC$ and the common Center of Gravity *C* wou'd quiesce. So that in equal Motions towards contrary Parts, whether the

Bo-

Bodies go from, or come towards one another, the common Center of Gravity is still at rest.

But if the Motions were so adjusted to each other, that $A : B > gC : dC$, or $A : B > fC : eC$, that is, $A \times dC > B \times gC$, or $A \times eC > B \times fC$; then some Point as k between e or d and C , may be taken so, that $A \times dk$ may $= B \times gk$; and then 'tis plain, that the Point k is the common Center of Gravity. Therefore in this Case, the common Center of Gravity has mov'd from the Point C to the Point k . So likewise it might be shown, that if $A \times dC < B \times gC$, or $A \times eC < B \times fC$, then the common Center of Gravity wou'd be in some Point as h between g or f and C , and so to have mov'd the Space Ch . So that 'tis most clear, that in unequal Motions towards contrary Parts, the common Center of Gravity moves in the same Right Line together with the Bodies themselves. Again, if the Bodies move towards the same Parts with any Motions whatsoever, it is manifest, that the common Center of Gravity, must move also. Let the Bodies A and B whose common Center of Gravity is C , move from the the Places A and B , into the Places e and g ; I say, the common Center of Gravity cannot be now in the Point C . For if it were, then it wou'd be $A : B :: gC : eC$, the Bodies being in e and g . But it is $A : B :: BC : AC$, the Bodies being in A and B . Therefore it wou'd be $gC : eC :: BC : AC$, that is $gC : BC :: eC : AC$, which

is impossible. Therefore the common Center of Gravity is not in C, but into some Point as *b* taken so, that it may be $gh : eh :: BC : AC$, or $gh : BC :: eh : AC$. And that it must necessarily be in some Point, as *b* between C and B, is too plain to need any Proof. And further, 'tis evident enough, that the Motion of the common Center of Gravity (whenever it moves, in any of these Cases) is uniform, or that it describes, equal Spaces in equal Times. For the Motion of the Bodies is uniform (by the Supposition) and the common Center of Gravity divides the Distance between the Bodies ever in the same *Ratio*; therefore as the Spaces described by the Bodies themselves in equal Times, will be equal, so also will those describ'd by the common Center of Gravity.

Hitherto we have consider'd the Bodies only as moving in one and the same Right Line. But if they move in different Right Lines (either parallel, or enclin'd to one another) the Motion of the common Center of Gravity may still be shewn to be perform'd in a Right Line, and to be uniform; for the Demonstration of which I refer the Reader to Mr. Keil's foremention'd Book. So also if the Bodies mov'd in different Planes, and it were ever so great a Number of Bodies, the Motion of whose common Center of Gravity was consider'd.

The

2. The next Thing to be clear'd, is, That when Two Bodies move either towards the same or towards contrary Parts, the Sum or Difference of their Motions is the same as if they were both carried with the Velocity of their common Center of Gravity.

This *Prop.* may be easily demonstrated, but I will at present give an Investigation of it. And the doing it in one Case alone, when the Bodies move towards the same Parts, will be Direction enough to the industrious Reader to do it in the other Case himself. In (*FIG. XXI.*) Let the Bodies A and B move from the Places A and B into the Places *e* and *g*, and their common Center of Gravity from C into *h*, at the same time. Let $AB=b$, $AC=c$. Then $BC=b-c$. Let $Ch=n$. $Ae=d$. Then $Bb=b-c-n$. and $eC=c-d$. and $eh=c-d+n$.

When the Bodies are in *e* and *g*, the Value of *gh* will be found $\frac{cb-ca-db-dc-nb-nc}{c}$, from the Nature of the common Center of Gravity,

Therefore $Bg (=gh-Bb) = \frac{dc+nb-db}{c}$. But the

Sum of the Motions of the Bodies is $=A \times Ae + B \times Bg$, that is, $A \times d + \frac{B \times dc + B \times nb - B \times db}{c}$. And

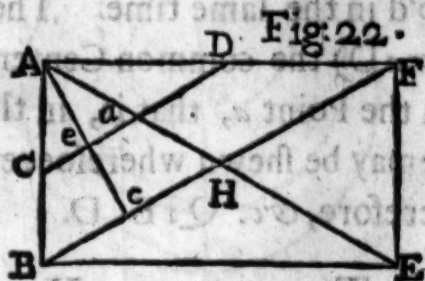
the Motion of both carried with the Velocity of the common Center of Gravity is $=A \times Ch + B \times Ch$, that is, $A \times n + B \times n$. And (multipling both Parts by *c*) we shall find $A \times nc + B \times nc = A \times dc +$

$B \times dc + B \times nb - B \times db$ (because $A \times c = B \times b - B \times c$, from the Nature of the common Center of Gravity, and consequently $A \times nc = B \times nb - B \times nc$, and $A \times dc = B \times db - B \times dc$.) The Result therefore is that the Motions are the same, as if both Bodies were carried with the Velocity of the common Center of Gravity. Q: E: I.

And now from both these *Props.* together arises a Demonstration of the Truth of this Third *Cor.* For (by the Second *Cor.* the Sum of the Motions towards the same Quarter remains the same, after the Bodies have struck one another, that it was before. But it has been shewn, that the Sum of the Motions either before or after the Stroke is the same as if the Bodies had been carried with the Velocity of the common Center of Gravity. From whence it may certainly be inferr'd, that the Velocity of the common Center of Gravity, is the same before as after the Collision of the Bodies. But if the common Center of Gravity moves at all before the Bodies meet and strike, it moves uniformly in a right Line, therefore after the Bodies have met and struck, it will move uniformly in a right Line also: As on the other Hand, if it be at rest before the Stroke it will easily be shewn to be at rest afterwards. Therefore the common Center of Gravity does not change its State of Rest or uniform Motion in a rectilinear

neal Direction by any Action of Bodies upon one another. $Q:E:D$.

And thus much for the Illustration of the *Coroll.* But before I pass to the next, (being now upon this Head) I'll add Two or Three *Props.* relating to the Motion of the common Center of Gravity.



T H E O R. I.

If Two Bodies equal in Magnitude, move in the Sides of a Parallelogram with Velocities proportional to those Sides respectively; their common Center of Gravity shall still be found in the Diagonal.

LET the Bodies (*FIG. XXII.*) move in the Lines FA , BA , from the Points F and B towards A . Draw the Diagonals AE , BF , cutting each other in H , and any other Line as CD parallel to the Diagonal BF , intersecting the Diagonal AE in the Point a . When the Bodies are in the Points B , F , 'tis plain, that their common Center of Gravity is in H ; for the Bodies

are

are equal by the Hypothesis, and BH is $= HF$ (from the Elements of the common Geometry.) Likewise because the Velocities of describing the Sides of the Parallelogram are suppos'd proportional to those Sides respectively, therefore (CD being parallel to BF) when one of the Bodies is in D , the other will be in C , for $CB : DF :: AB : AF$; consequently BC and DF will be describ'd in the same time. Therefore (because $Ca = aD$) the common Center of Gravity will be in the Point a , that is, in the Diagonal. The same may be shewn wheresoever the Bodies are. Therefore, &c. $Q : E : D$.

T H E O R. II.

If the Bodies were unequal in Magnitude; the Velocities with which they move being as the Sides of the Parallelogram respectively; the Path of the common Center of Gravity (which will not be the Diagonal) is easily determin'd.

LET the Ratio of the Magnitude of the Bodies be what it will; then from A (*FIG. XXII.*) draw the Line Ac , to cut the Diagonal BF in the Point c in such a Manner, that the Segments Bc , and cF may be reciprocally as the Magnitudes of the Bodies plac'd in B and F . 'Tis evident, that the common Center of Gravity shall describe the Path Ac , while the Bodies them-

in the outer and greater Circle. And let these Bodies be called A and B respectively. Then by the Hypothesis 'tis $A+B:B::AB:AC$. Therefore (by Division) $A:B::BC:AC$; consequently when the moving Body is in B, the common Center of Gravity is in C. But (from the Nature of the Circle) $AB:AC::AD:AE$ (because the Angles at D and E are right, and that at A is common, and to the Triangles ADB and AEC are Similar) also 'tis $BC:AC::DE:AE$. But $BC:AC::A:B$, therefore $DE:AE::A:B$. Therefore when the moving Body is in D, the common Center of Gravity is in E the Periphery of the inner Circle, and so it will be always found in the same. Q: E: D.

THEOR.

T H E O R. IV.

If Three Circles touch'd one another in the same Point A, (FIG. XXIII.) it might be shewn, that if the Body A mov'd in the Periphery AFG, and the Body B in the Periphery ADB, and the Sum of the Two Bodies were to the Body B, as the Difference of the Diameter of the outer and inner Circles, to the Difference of the Diameters of the middle and inner Circles. Also if the Velocities of the Bodies were to one another directly as the Peripheries which they describe; I say then, that the common Center of Gravity shall describe the Periphery of the middle Circle, viz. AEG.

FOR if the Velocities be as the Peripheries, they are also as any Two similar Arches FG and DB cut off by any right Line AFD drawn at Pleasure; therefore in the same Time that one Body describes the Arch BD, the other will describe the Arch FG; that is, when one is in D, the other will be in F in the same right Line AFD. Also because (by the Hypothesis) 'tis $A+B : B :: BG : GC$, by division 'tis $A : B :: BC : GC$. But (from the Nature of the Circle and the Similarity of the Triangles) 'tis $BC : GC :: DE : EF$, therefore $A : B :: DE : EF$.

And

And therefore the common Center of Gravity is in the Point E of the middle Circle.

C O R.

Hence if the Velocities of the Body were reciprocally proportional to their Magnitudes, then AB the Diameter of the outer Circle, would be divided Harmonically, in the Points, A, G, C, and B, and *Vice Versâ*. If the Lines BC, GC, AB, AG be in musical Proportion, then the Velocities of the Bodies are reciprocally proportional to their Magnitudes. Let the Velocity of the Body B be put $=V$, and that of the Body A be put $=v$. Then (by the Hypothesis of the Theorem) 'tis $V : v :: AB : AG$, for the Velocities which are as the Peripheries, are also as the Diameters. But (because of the common Center of Gravity) 'tis $A : B :: BC : GC$. Therefore if it be $V : v :: A : B$, (that is, if the Magnitudes of the Bodies are reciprocally as their Velocities) then it follows, that $AB : AG :: BC : GC$, and consequently $AB \times GC = AG \times BC$, which is the Property of a Line cut in musical proportion. The Line AB therefore will be divided Harmonically,

THEOR.

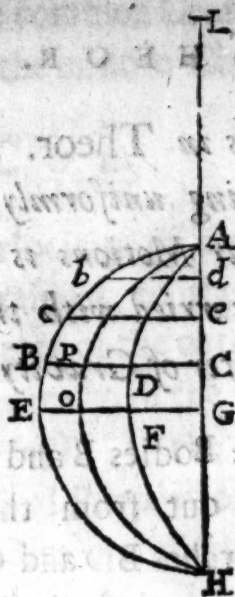
T H E O R. V.

Supposing all as in Theor. 4. I say, that Two Bodies moving uniformly in Circles, the Sum of their Motions is the same, as if both were carried with the Velocity of the common Center of Gravity.

WHile the Bodies B and A (*FIG. XXIII.*) Setting out from the Points B and G, describe the Arches BD and GF; the common Center of Gravity describes the Arch CE, as has been already demonstrated. The Sum of the Motions of both therefore, carried each with their own Velocities, is $= B \cdot BD + A \cdot FG$; and the Sum of the Motions, if both were carried with the Velocity of the common Center of Gravity, would $= A \cdot EC + B \cdot EC$. Now I say, that $A \cdot EC + B \cdot EC = B \cdot BD + A \cdot FG$. For (from the Nature of the common Center of Gravity) 'tis $A : B :: BC : GC$, or (because $BC = AB - AC$, and $GC = AC - AG$) $A : B :: AB - AC : AC - AG$. From whence $A \cdot AC - A \cdot AG = B \cdot AB - B \cdot AC$; and $A \cdot AC + B \cdot AC = B \cdot AB + A \cdot AG$. But because the Arches DB, EC, FG are similar, they are porportional to their respective Diameters. Therefore $A \cdot EC + B \cdot EC = B \cdot DB + A \cdot FG$. Q. E. D.

T H E O R.

Fig. 24.



T H E O R. VI.

If a Body moves in the Curve of a Conick Section, whilst another moves in the Ax of the same, and their Motions be adjusted so, that while one describes any Portion of the Curve, the other describes the correspondent Absciss, their common Center of Gravity shall always describe one of the Conick Sections.

C A S E I.

Suppose the Curve ABE (FIG. XXIV.) were a Circle, whose Radius is AG. Let one Body be imagin'd to describe AB or AG in the Ax, while the other describes AB or AE in the

the Circumference; and in the *Radius* EG, let some Point as F be taken, dividing the Line EG, in the *Ratio* of the Magnitudes of the Bodies. I say, that an Ellipse, whose Transverse Ax is equal to the Diameter of the Circle, and whose Semi-conjugate is equal to FG (that is, supposing EGA a Right Angle) shall be the Path describ'd by the common Center of Gravity. For, conceive an Ellipse passing thro' the Points A, F, H; and draw the Ordinate BC, *cc*, &c. cutting an Ellipse in D, *n*, &c. Then (from the Nature of the Circle $EG^2 = AG \times GH$, and $BC^2 = AC \times CH$. And from the Nature of the Ellipse, $FG^2 : DC^2 :: AG \times GH : AC \times CH$. From whence $EG^2 : FG^2 :: BC^2 : DC^2$; and $EG : FG :: BC : DC$, and (by Division) $EF : FG :: BD : DC$, and so, the Curve of the Ellipse will divide all the Ordinates of the Circle in the same standing *Ratio* of EF to FG. Therefore if $EF : FG ::$ as the Body in G : to the Body in E, then their common Center of Gravity will be in F; and when the Bodies are in B and C, 'twill be in D; when they are in *c* and *e*, 'twill be in *n*. Therefore the common Center of Gravity will describe the Curve of this Ellipse. Q: E: D.

C A S E II,

If the Curve OAH were an Ellipse, having the Transverse Ax AH; it might be shewn, that the common Center of Gravity would de-

scribe another Ellipse, as ADF, having the same Transverse with the former. For (from the Nature of these Curves) $OF : FG :: pD : DC$. So that a Body moving in the Ellipse AOH, while another moves in the Ax; dividing the Semi-conjugate in the Point F, in the *Ratio* of the Magnitudes of the Bodies; the common Center of Gravity shall describe the Ellipse AFH.

C A S E III.

If one Body describ'd the Circle AEH, and another the Ellipse AFH; then dividing the Line EF in the Point O, in the *Ratio* of the Magnitudes of the Bodies; it may be prov'd after the same Manner, that the common Center of Gravity shall describe the Ellipse AOH, passing thro' the Point O, and having the same Transverse Ax AH. For it may be demonstrated as above, that $EF : FG :: BD : DC$. And that $OF : FG :: pD : DC$. Therefore $EF : OF :: BD : pD$; and (by Division) $EO : OF :: Bp : pD$. So that if when the Bodies are in E and F, the common Center of Gravity be in the Point O; then when they are in B and D, it shall be in the Point p. Therefore the common Center of Gravity shall describe the Ellipse AOH.

C A S E IV.

If the Curve ABE were a Parabola, whose Ax is AG, and Ordinate EG, it may be demonstrated

strated, that the common Center of Gravity shall describe another Parabola (as ADF, or ApO) having the same common Ax. For (from the Nature of these Curves) 'tis $EG^q : BC^q :: AG : AC$. And $FG^q : DC^q :: AG : AC$, therefore $EG^q : BC^q :: FG^q : DC^q$, and $EG : BC :: FG : DC$, and alternately $EG : FG :: BC : DC$, and (by Division) $EF : FG :: BD : DC$, and so 'tis every where. Consequently if EG be divided in F in the *Ratio* of the Magnitudes of the Bodies, the common Center of Gravity shall describe the Parabola ADF, or (which is all one) if upon the Ax AG and with a Parameter = $\frac{FG^q}{AG}$ we describe a Parabola, this Curve shall be the Path of the common Center of Gravity.

C A S E V.

In like manner it might be shewn, that if the Three Curves ABE, ApO, ADF were Parabola's; the Bodies describing the outer and the inner Parabola's, the common Center of Gravity wou'd describe the middle one, viz. ApO; that is, supposing their Motions to be so adjusted as was mention'd in the *Theorem*, and the Interval EF to be divided in the Point O, in the *Ratio* of the Magnitudes. For (in these Curves also) $EO : OF :: Bp : pD$.

C A S E. VI.

And, Lastly, If the Curve ABE were an Equilateral Hyperbola, having the Transverse AL, and Ordinate EG; it may be demonstrated, that the common Center of Gravity shall describe another (but not an Equilateral) Hyperbola, having the same Transverse AL with the former; or, which is all one, that such on Hyperbola as this shall be the Track of the common Center of Gravity, as the Bodies move one in the Curve, and the other in the Ax, according to the Intent of the *Theorem*. For in the Equilateral Hyperbola $EG^2 = LG \times AG$, and $BC^2 = LC \times AC$; and in the other Hyperbola (from the *Conick Elements*) 'tis $FG^2 : DC^2 :: LG \times AG : LC \times AC$. So that in these Curves also 'twill be concluded, that $EF : FG :: BD : DC$; and from thence the Consequence (so often repeated) with Reference to the common Center of Gravity, is plain and clear. And the same may be shewn here of Three Hyperbola's, as was before of Three Parabola's, or of Two Ellipses and a Circle.

There are some of the *Mechanick* Curves, in which I cou'd apply this Matter yet farther; but I must observe just Bounds, and (as 'tis now time) leave this Subject.

C O R.

'Tis evident from these several Cases, that the common Center of Gravity describes a *Conick Section* of the same Family with that which the Bodies themselves describe; if at least (as to the first *Case*) one may be allow'd to make the Circle and Ellipse akin to one another; and say, that a Circle is an Ellipsis, whose Transverse and Parameter are equal to one another.

Fourth COROLLARY to the general Law of Motion.

The Motions of Bodies (included in any given Space) are the same with respect to one another, whether that Space be at Rest, or whether it moves uniformly in a Right Line, without any circular Motions.

THIS is easily demonstrated from the Second *Corollary* to the *General Laws of Motion*; and particularly from the Deduction made from thence, about the relative Velocity being ever proportional to the Magnitude of the Stroke. If the relative Velocities continue ever the same (as they do, whether the Space, in which they are contain'd, moves, or is at rest) then the Magnitudes of the Strokes which the Bodies make upon one another, are the same

still, and so their Motions are the same with respect to one another.

Fifth COROLLARY to the general Laws of Motion.

If Bodies are mov'd any how amongst themselves, and are urg'd by equal accelerating Forces, that act with parallel Directions, their Motions will proceed in all Respects, as if they were not influenc'd at all by those accelerating Forces.

FOR those Forces acting equally (with respect to the Quantities of the Bodies to be moved) and in parallel Directions also; they will (by the Second general Law of Motion) as to the Point of Velocity, equally move the several Bodies, and consequently never alter their Motions and Positions with Respect to one another.

Having thus endeavour'd to make some Applications of those most useful and comprehensive *Laws of Nature* laid down by the Great and *Illustrious Author*; I shall go on to his *Doctrine of Centripetal Forces*, and explain some of the wonderful *Theorems* he has discover'd upon that Head.

The End of the First Part.

OF THE Centripetal Forces, AND

Velocities of Bodies, Moving in Curvilinear Figures.

PART II.

DEFINITION I.

Centripetal Force is that, by which a Body is drawn, or impell'd, or by any other Means (howsoever) made to tend to a certain Point, as a Center.

OF this kind is the Force of Gravity, by which a Body tends to the Center of the Earth; the Magnetick Force by which a Body tends to the Center of the Loadstone; and that Force (whatever it be) by which the Planets are continually with-held from going on in Right Lines, and forced to move in Curves,

DEFINITION II.

The absolute Quantity of the Centripetal Force, is the Measure of it, greater or less, in Proportion to the Efficacy of the Cause, propagating it a Centro ad circumferentiam.

THUS in one attractive Body, the Power of Attraction is stronger and greater than in another. The Magnetick Vertue is more vigorous and forcible in one Load-stone, than in another. And (in the greater Systems of Matter) that Force, by which the same Bodies, at the same Distance from the Centers of the Planets, is urged towards them, is stronger in one Planet than another. Thus (as we shall see afterwards) the Weight of the same Body at any Distance from the Center of the Sun is greater than its Weight at the same Distance from the Center of *Jupiter*; and this greater than that, at the same Distance from the Center of *Saturn*; and this greater than that, at the same Distance from the Center of the Earth.

DEFINITION III.

The accelerating Quantity of the Vis Centripeta, is the Measure of it, with respect to the Velocity which it generates in a given Time.

EX. gr. The Vertue of the same Magnet is greater at a less Distance, and less at a greater Distance. The gravitating Force of Bodies, is greater in the Valleys and depress'd Places of the Earth, than upon the Tops of high Hills and Mountains; and is yet greater in these elevated Places, upon the Surface, than at greater Distances from it. But in equal Distances from the Earth, 'tis every where the same; for all descending Bodies, be they more heavy or light, greater or smaller, abating the Resistance of the Air, are equally accelerated. The manifold Experiments made upon *Pendulums* have sufficiently prov'd these Things to be true in Fact.

DEFINITION III.

The Quantitas Motrix of the Centripetal Force, is the Measure of it, proportional to the Motion which it generates in any given Time.

Such is more or greater Weight in a greater Body, and a less in a less Body; and greater Weight

Weight in the same Body near the Earth, and less at a farther Removal upwards. This Force is the *Centripetentia*, or Propension of a Body towards a Center, or (as one may properly enough say) the *Pondus* of it; and 'tis always known (that is, the Magnitude of it is measured or found) by that of an equal and contrary Force, such as is sufficient to hinder the Descent of the Body.

E X P L I C A T I O N.

These Quantities of the Centripetal Force Mr. *Newton* calls by these Names, and, for Distinctions sake, considers them as referring to Bodies, to the Places of Bodies, and to a Center of Forces. Namely, the *Vis Motrix* relates to Bodies themselves, as the Endeavour and Propension of the whole (compounded of the Propensions of all the Parts) towards a Center. The *Vis Acceleratrix* respects the Place of a Body, as a certain Force or Efficacy (propagated from a Center, and diffused thro' all the circumjacent Regions) to move Bodies that are in those Places. The *Vis Absoluta* respects the Center it self, as some way or other a Cause, without which the *Vires Motrices* are not propagated thro' the Regions about it. What the Cause and Origin of Centripetal Force is, is not necessary to be determin'd here; 'tis sufficient that there is such a Thing in Fact. As for Mat-
ters

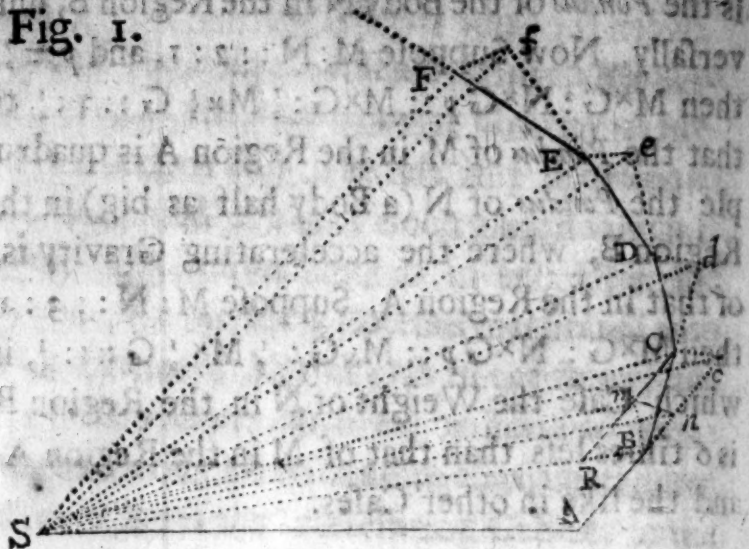
ters of pure Physical Consideration (such as are the Causes and Subjects of these Forces) 'twas Foreign to our Author's Purpose and Design to enter upon any Discussion of them in this Place. But here we may take Notice of an useful Comparison between the *Vis Acceleratrix*, and what we call *Velocity*. For the *Vis Acceleratrix* bears the same Relation to the *Vis Motrix*, that *Velocity* does to *Motion*; and as the Quantity of *Motion* arises from the Multiplication of the *Velocity* into the Quantity of *Matter*, so does the *Vis Motrix* from the Multiplication of the *Vis Acceleratrix* into the Quantity of *Matter*. For the Sum of the Actions of the *Vis Acceleratrix* upon all the Particles of a Body, is the *Vis Motrix* of the whole; as the Sum of the *Momenta*, arising from the Multiplication of the *Velocity* into all the Particles of a Body, is the compleat *Momentum*, or Quantity of *Motion* of the whole: So that, as in Bodies of different Magnitudes, when the *Velocity* is the same in all of them, the Quantities of *Motion* are proportional to the Magnitudes or Bulks; so in all Bodies, when the accelerating Gravity is the same (that is, near the Surface of the Earth) the *Gravitas Motrix*, or the *Pondus*, is proportional to the Quantities of *Matter*, that is (in Homogeneous Bodies) to the Magnitudes of them. But then, ascending higher into the Regions where the accelerating Gravity is less, the

Pon-

Pondus of the Body is lessen'd in proportion to that, even as the *Momentum* or Gravity of Motion in a Body is said to be less, in proportion to the Diminution of the Velocity with which it moves. But universally, as in one Case the *Momenta* are as the Velocities multiplied into the Quantities of Matter; so in the other, the *Vires* or *Gravitates Motrices*, or *Pondera* are as the accelerating Gravities into the Quantities of Matter. Thus if the accelerating Gravity in the Region *B* be twice less than it was in the Region *A*, then the *Pondus* of the same Body *M* shall be in the Region *B*, but half what it was in the Region *A*; but if the Body *N* be twice or thrice less than the Body *M*, then the Weight of *N* will be 4 or 6 times less in the Region *B*, then it was in *A*.

Suppose *G* = the accelerating Gravity of the Body *M* in the Region *A*, and, the Quantity *p* denoting either an Integer or a Fraction, *Gp* shall express the accelerating Gravity of *M* in the Region *B*, according as it is greater or less there then in the Region *A*. And because the accelerating Gravities of all Bodies in the same Places or at the same distances, are equal; therefore these Quantities *G* and *Gp* shall express the accelerating Gravities of the Body *N*, Posited at the same Distances, in the Regions *A* and *B*, with the Body *M*. Therefore *M* × *G* is the *Pondus* of the Body *M* in the Region *A*, and *N* × *Gp* is

Fig. I.



Prop. I. Theor. I.

The Area's which revolving Bodies describe (by Rays drawn to an immovable Center of Force) do lie in immovable Planes, and are proportionabce to the Times in which they are describ'd.

LET SABCDEFS be (FIG. I.) any Polygon, and the Lines AS, BS, &c. drawn to the fix'd Point S. Produce the sides AB, BC, CD, DE, EF to the the Point *c*, *d*, *e*, *f*, in such a Manner that AB may = B*c*, BC = C*d*, CD = D*e*, DE = E*f*. From the Point C draw CR to cut BS in R, and let it be parallel to *c* B, and C*c* parallel to BS; so likewise draw D*d* parallel to SC, and E*e* to SD, and F*f* to SE. The Triangles SBC, SB*c* are equal being upon the same Base SB, and between

tween the same Parallel's SB and Cc . Also since $AB=Bc$, the Triangles ASB , $BS c$ are equal, therefore $ASB=BSC$. So in like manner 'twill be prov'd, that the Triangle CSD is $=CSd$ being upon the same Base CS , and between the same Parallel's CS , Dd ; but because $BC=Cd$, the Triangle $BSC=CS d$, therefore $BSC=CSD$, and consequently $ASB=CSD$; and by the same way of arguing we shall conclude all the Triangles of the Polygon to be equal to one another, viz. $ASB=BSC=CSD=DSE=ESF$, &c. Now I shall demonstrate that in equal Times, a Body (urg'd by a centripetal Force tending to the Center S) will describe these equal *Area's*. For let T express any Portion of Time divided into several equal Parts express'd by $A. B. C. D. E.$ &c. and let us imagine a Body plac'd in the Line AB at the Point A , and to be carried by such a Force that in the first part of time A it wou'd describe the Line AB . It's plain (by the first Law of Motion) that if the Body were not otherwise hinder'd in the 2d part of Time B , it wou'd describe the Line $Bc=AB$, and so the *Area's* ASB , $BS c$ wou'd be equal. But when it is come to the Point B imagine the centripetal Force to exert it self and begin to cause the Body to tend towards the Center S (which we will suppose to be the Center of Forces) and let this centripetal Force be such, that if the Body were entirely acted by that, it wou'd make it describe the

the

the Line RB in the Time B. Here then the Body at the Point B is drawn by two Forces, the one its innate Force whose Direction is from B to *c*, the other the Centripetal, whose Direction is from B to R. Therefore (by *Corol.* 1. of the Laws of Motion) the Body will move in the Line BC the Diagonal of the Parallelogram RBC*c*; and at the end of the Time B it will be found in the Point C, ever in the same Plane with the Triangle ASB. In like manner in the 3d part of Time C the Body will describe the Line CD the Diagonal of the Parallelogram of which the Triangle CD*d* is the $\frac{1}{2}$: And in the 4th part of Time 'twill describe the Diagonal DE, and so of the rest: So that by the Conjunction of these two Forces it will describe the Periphery of the Poligone ABCDEF. Now the Triangles ASB, BSC, CSD, &c. were proved to be equal; but they are describ'd in the equal times A, B, C, &c. therefore equal *Area's* are describe in equal Times; and consequently Time A: Time $\overline{A+B} :: \text{Area ASB} :: \text{Area } \overline{\text{ASB+BSC}}$, and Time A: Time $\overline{A+B+C} :: \text{Area ASB} : \text{Area } \overline{\text{ASB+BSC+BSD}}$, and so on; that, is the *Area* describ'd are as the Times of Description. Now if the Lines AB, BC, CD, &c. the Cases of the Triangles be diminish'd, and their Number augmented infinitely, the Parameter of the Poligone will become a Curve Line, and so the Centripetal Force (by which the Body is drawn out of the

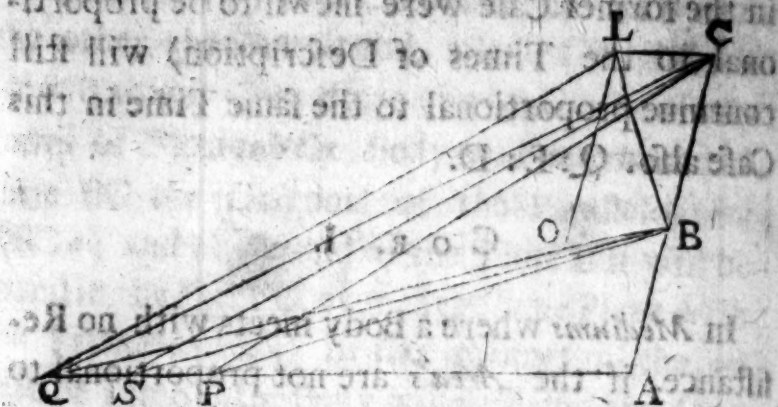
the

the Tangents) will act incessantly, and without Intermision. And any *Area's* describ'd (which in the former Case were shewn to be proportional to the Times of Description) will still continue proportional to the same Time in this Case also. $Q : E : D$.

C O R. I.

In *Mediums* where a Body meets with no Resistance, if the *Area's* are not proportional to the Times, the Forces do not tend to the common Concourse of the Rays. I say, in non-resisting *Mediums*: For in *Mediums* where a Body meets with Resistance, and so is any way hinder'd in its Motion, the *Area's* may possibly not be proportional to the Times, and yet, the Forces may tend to the common Concourse of the Rays. *Ex. gr.* Suppose in (*FIG. 1.*) the Body to have describ'd the Line AB in the Time t , and whereas (if not otherwise hinder'd) it wou'd describe $Bc = AB$ in the next equal Portion of Time, imagine it to meet with some Resistance, so as to describe no more than Bn in the same Time; then the Triangle $SBn = SBm$ will be less than SBA ; so that the centripetal Force acting as before in the Case where we suppos'd no Resistance, the *Area's* will not now be proportional to the Times, and yet the Forces tend to the Point S the common Intersection of the Rays.

Fig. 2.



C O R. II.

In all *Mediums* whatsoever, if the Description of the *Area's* be accelerated (that is, if an equal *Area* be describ'd in a less Portion of Time, or a greater *Area* in an equal Time) then the Forces do not tend to the common Concourse of the Rays, but decline from thence in *Consequentia* (that is, to the Parts towards which the Body is moving.

For (*FIG. II.*) AB being $= Bc$, the *Area's* ASB, BSc , taken about the same Point S , will ever be equal. But if we took a Point as P between S and A , the *Area's* ASB wou'd be greater than the *Area* BPc . On the other Hand, if we take a Point as Q on the other Side of S , the *Area* BQc will be greater than ASB . Therefore if

when

when in the first Portion of Time the *Area* ASB was describ'd, in the 2d Time a greater *Area*, as BQL (=BQc) be describ'd; 'tis certain, that the centripetal Force must have acted with a Direction, as LC parallel to B_oQ, that is, that the Forces now tend to the Point Q. The same may be shewn for any other Point taken in the same Right Line beyond Q; if the Direction of the *Area's* be accelerated, the Tendency of the Forces will still be in Consequence of the Plan

THEOR. II.

Every Body moving in a Curve, and (by Rays drawn to a certain Point, either immovable, or going on uniformly in a Right Line Motion) describing *Area's* about that Point proportional to the Times, is urged by a Centripetal Force tending to that same Point. (See FIG. I.)

CASE I.

FOR every Body moving in a Curve, is carried out of its Right Line Course by some Force acting upon it, by the first *Law of Motion*. And that Force by which the Body is made to defect from its Rectiline Course, and to describe the little equal Triangles SAB, SBC, SCD, in equal Times, about the immovable Point S, must act in the Point B with a Direction

rection parallel to the Line Cc , by the Second *Law of Motion*: That is, it acts with the Direction BS . So in the Point C it acts according to a Line parallel to Dd , that is, by the Line CS , for the same Reason: And so 'tis every where. Therefore it always acts according to Lines that tend to the immovable Point S . Q.E.D.

C A S E II.

If the Plane in which the Body describes a Curvilinear Figure (and consequently the Center or Point to which the Forces tend) moves uniformly in a Right-line Direction; 'tis all one as to the Effect (by *Coroll. 4.* to the general *Laws of Motion*) whether it does so, carrying the Body and All along with it, or whether it be absolutely at rest. So that either way the Truth of the Proposition is manifest. Q.E.D.

S C H O L.

'Tis possible that a Body may be urg'd by a Centripetal Force, compounded of several other Forces; about which Composition of Forces we have spoke sufficiently already in the first Pages of this Book. Now always in such Cases as this, the Sense of the foregoing *Theorem*, is, that the Force which is compounded of all the rest, is that which tends to the Center. *Ex. gr.* Suppose in *FIG. II. Pag. 7.* where the Force acting with the Direction CB is shewn to be equi-

equivalent to the Forces acting with the Directions CD, CE, CF. We are to conceive the compound Force, whose Direction is CB to tend to the Center; so that the Center of the Forces, where all the Rays meet, is somewhere or other in the Line CB (produced at least.

'Tis further to be noted, that if there be any Force that acts with a Direction perpendicular to the Superficies describ'd by the Body, that this Force may be altogether left out of the Account of the Composition of the Forces. For tho' it makes the Body to decline from the Plane of its Motion, yet it tends no way to alter the Magnitude of the Surface describ'd, neither as to the encreasing or lessening of it, and may therefore be neglected.

THEOR. III.

Every Body (which by a Ray drawn to the Center of another Body (howsoever mov'd) describes Area's about that Center proportional to the Times) is urged with a Force compounded of the Centripetal Force tending to that other Body, and of all the accelerating Force with which that other Body it self is urg'd.

LET the Body M describe *Area's* proportional to the Times, about the Center of the Body N; and let the Body N be urged with

an accelerating Force, as $+A$. Suppose now both Bodies to be drawn in a parallel Direction by a new Force equal and contrary to the former, which put $= -A$. Then, notwithstanding, the Body M will describe *Area's* proportional to the Times, as it did before the Action of this new Force, with respect to N, (by *Cor. 5. to the Laws of Motion*;) but the Force $-A$ will destroy the Force $+A$ in the Body N (which will now therefore either quiesce, or move uniformly in a Right Line (by the first *Law of Motion*.) But also because the Body M is influenced by this new Force $-A$, therefore the Force by which it at first tended to the Body N being call'd C, the Force by which it is now urged shall $= C - A$; which Force (by *Theor. II.*) shall still tend to the Center of the Body N, because we have shewn, that the *Area's*, with respect to that Center, are still proportional to the Times. So that every Body, &c. Q: E: D.

C O R. I.

Hence if one Body, with a Ray drawn to another, describes *Area's* proportional to the Times; and if from the whole Force (whether a simple or compounded one) with which the former Body is urged, we subtract the whole accelerating Force with which the latter Body is urged; all the remaining Force by which the
the

the former Body is urged will tend to the latter Body, as its Center. This is plain from that Demonstration; for the resulting Force is = C-A. the Centripetal Force of the

C O R. II.

If the *Area's* be not accurately, but only nearly proportional to the Times, then the remaining Force will not accurately, but nearly tend to the other Body. And the Converse of this is true also.

C O R. III.

If a Body with a Ray drawn to another Body describes *Area's*, which compar'd with the Times, are very unequal; and that other Body is either quiescent, or moves uniformly in a Right Line: Then, either there is no Action of the Centripetal Force tending to that other Body as its Center, or else its mix'd and compounded with some very strong Actions of other Forces: And the whole Force (compounded of all, if there are many) is directed to another Center, either a movable or an immovable one, about which the Description of the *Area's* is equable and regular. And the same holds, when that other Body is moved even with any sort of Motion; provided the Centripetal Force (that which we here say is directed to

another Center) be taken for the Remainder, after the Subtraction (of the whole accelerating Force that acts upon the latter Body) from the real Centripetal Force of the former Body.

The Reason of all which is easily deducible from the Second *Theorem* (foregoing) in Conjunction with this Third. For where the *Area's* describ'd are proportional to the Times, there the Action of the Centripetal Force tends to a Center, that is, either quiescent or moves uniformly in a Right Line: This is plain from *Theor. II.* Therefore if that Center be either quiescent, or moves uniformly in a Right Line; and the *Area's* describ'd be not proportional to the Times, 'tis a sure Proof that either the Action of the Centripetal Force does not tend to that Center at all, or else that 'tis confounded and disturb'd by the powerful Actions of other Forces. And if according to the Tenour of *Theor. III.* the Center here spoken of be another Body, the Reason of the present *Corollary* will be sufficiently clear and manifest.

LEMMA

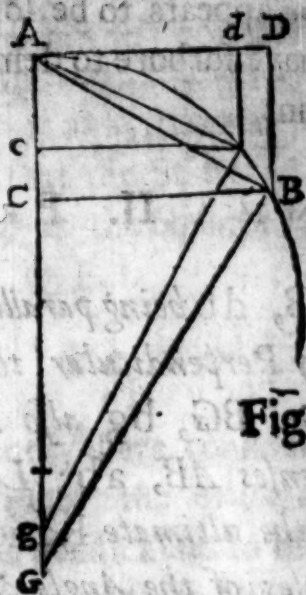


Fig 3.

LEMMA I.

Let AB be any Arch of a Curve, (FIG. III.) the Right Line AB its Subtense, and AD its Tangent equal and parallel to the Ordinate CB . When the Points A and D come infinitely near together, the Arch AB , the Subtense AB and the Tangent AD will be equal to one another; or in the Newtonian Style, the last Ratio of the Arch, the Chord and Tangent will be a Ratio of Equality. Each one of these Lines therefore may be used or taken for the other in all Argumentations about ultimate Ratio's; that is, when these Quantities are consider'd as vanishing upon the infinitely near Approach of the Points A and B together.

This

This Lemma appears to be so clear and evident, that I have forbore to demonstrate it upon that Account.

LEMMA II. PROB.

The Lines DB , db being parallel to AC , and consequently Perpendicular to AD ; drawing the Lines BG , bg also Perpendicular to the Subtenses AB , ab ; Let it be required to find the ultimate Ratio of DB , db (the Subtenses of the Angle of Contact) to the conterminal Arches AB , ab ; because of the Right Angles Abg , ABG , a Circle pass thro' the Points Abg , as also another Circle thro' the Points A, B, G ,

LET the Arch $AB = A$, $Ab = a$ $AG = 2R$; $Ag = 2r$. $AC = X$; $Ac = x$. $BC = Y$. $bc = y$, $AB = D$ and $Ab = d$. Now because of the Circles we have these Equations $YY = 2RX - XX$, and $DD = 2Rx - xx$ and $yy = 2rx - xx$ and $dd = 2dx - xx$; and dividing by $2R$ and $2r$ (which are each of them a standing Quantity) DD will always be as X , and dd as x . But when the Points A, B , and A, b approach infinitely near, the Quantity X becomes $\dot{x}$, and x becomes $\dot{x}$, and are the Subtenses of their respective Angles of Contact, also the Chord D is in this Case

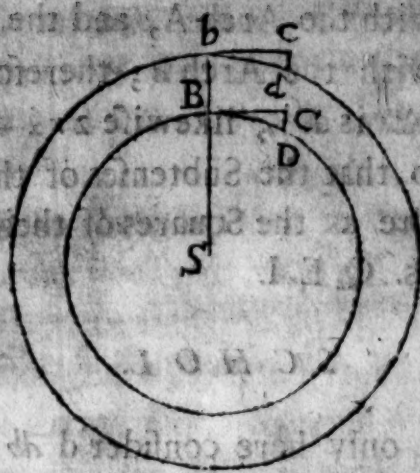
coincident with the Arch A , and the Chord a coincident with the Arch a ; therefore $2R\dot{x}$ is $= A^2$ and A^2 is as $\dot{x}$, likewise $2r\dot{x} = aa$, and aa is as $\dot{x}$, so that the Subtenses of the Angles of Contact are as the Squares of their conterminal Arches. Q. E. I.

S C H O L.

We have only here consider'd db and DB (the Subtenses of the evanescent Angles of Contact) as Perpendiculars to the Tangent AD ; but it will be the same thing if we suppose them to be inclined, for let AD and BD make any given Angle, then the ultimate Ratio of BD to bd will be the same as before, and consequently the same with that of ABq to abq ; If the Angle D be not given, yet the Angles D, d will always converge to, and come nearer an Equality, than by any Difference assignable, and consequently will at last be equal; therefore the Lines BD, bd are in the same Ratio to one another as before.

THEOR.

Fig. 4.

**THEOR. IV.**

Bodies describing different Circles with an equable Motion; their Centripetal Forces tend to the Centers of those Circles; and to the Law of those Forces may also be discover'd. (FIG. IV.)

1. **T**HAT the Centripetal Forces tend to the Centers of these Circles, is most evident. For since the Motion in the Peripheries is supposed equable, therefore from the common Principles of Mechanicks, the Arches run will be as the Times in which they are run. But the Sectors (or *Area's* taken from the Centers of the Circles) are in Proportion as the Arches upon which they stand. Therefore those *Area's* are as the Times; and therefore (by *Theor. II.*) the Centripetal Forces tend to the Centers.

2. *Imag.*

2. Imagine the Bodies to describe the Arches BD, bd in the same time, and let CD, cd be drawn parallel to SBb , that is, perpendicular to the respective Tangents BC, bc . Now since 'tis by the Centripetal Forces that the Bodies are drawn out of the Tangents into the Circumferences, 'tis plain that those *Lineola* CD, cd (consider'd not as of any finite Length) will represent the Centripetal Forces: For they are the genuine Effects of those Forces, produced in the same time, and therefore are proportional to them. Again, because they are *Lineola Nascentes*, they will not differ from others that tend to the Centers of the Circles, whether the Centripetal Forces ought to be directed, by the former Part of this *Prop. III.*

3. Now to discover the Law of these Forces, let $S=R. SB=r. bd=A, BD=a. cd=\dot{x}$ (Subtense of the Angle of Contact, in the greater Circle) $CD=\dot{x}$ (Subtense of the Angle of Contact) in the less. By *Lem. II.* we have $2R \dot{x} = AA.$ and $2r \dot{x} = aa,$ so that $A^2 : a^2 :: 2R \dot{x} : 2r \dot{x},$ consequently $\dot{x} :$

$\dot{x} :: \frac{A^2}{R} : \frac{a^2}{r}.$ That is this *THEOR. viz.* The

Centripetal Forces are as the Squares of the Arches (describ'd in the same Time) divided by the respective *Radii.* *Q: E: I.* In order to the Investigation of the many noble *Covollaries* that flow from this *Theorem,* we are obliged to introduce these Symbols. $C =$ Centripetal Force.

Force. T = Time, V = Velocity in the outer Circle bd ; and c, t, u , to express the like Quantities in the inner Circle BD . By the Theorem 'tis $C : c :: \frac{A^2}{R} : \frac{a^2}{r}$; but because the Arches A, a , are describ'd in the same Time, therefore, by the Principles of *Mechanicks*, the Velocities are as the Spaces, that is, the Arches run; therefore $V : v :: A : a$, therefore $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$, that is **C O R. I.** The Centripetal Forces are as the Squares of the Velocities, divided by the Radii. || This is the Law for the Centripetal Forces, in the Case of infinitely small Arches describ'd in the same Time. Let T, t , express now the whole Periodick Times of describing the Circles, and let the Periphery of the outer Circle = P , and that of the inner = p , and all the rest as before. From *Mechanicks*, the Times of describing the Peripheries P, p , are in the Ratio compounded of the direct Ratio of those Peripheries (or the Space, run) and the Reciprocal of the Velocities, that is, $T : t :: P \times v : p \times V$; but $P : p :: R : r$ (the Peripheries are as the Radii) therefore $T : t :: R \times v : r \times V$, but also $V : v :: A : a$ (the Velocities in the equable circular Motions are eternally as the Arches describ'd in the same Time) therefore $T : t :: R \times a : r \times A$, and $T r A = t R a$, and (squaring) $T^2 r^2 A^2 = t^2 R^2 a^2$, therefore

A²

$\frac{A^2}{R} \propto \frac{c^2}{r} \propto \frac{t^2}{R}$, but (by the Theor.) $C : c ::$

$\frac{A}{R} : \frac{a}{r}$, therefore $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$, or, which is

all one, $C : c :: R \times t^2 : r \times T^2$, that is **Cor. II.** The

Centripetal Forces are in the *Ratio* compound-

ed of the direct *Ratio* of the *Radii*, and the re-

ciprocal *Ratio* of the Squares of the Periodick

Times. Which is the Law for the Centripetal

Forces, when we consider the whole Periodick

Times. Now, from these general Laws all the

others, relating either to Centripetal Forces

or Velocities, are easily deduced, making any

Supposition whatsoever. For, whatever the Pe-

riodick Times be, we have got a general *Theo-*

rem. for the Centripetal Forces in this last *Corol-*

lary: And as for the Velocities, they are ever

as the *Arches* A, a , describ'd in the same Time,

and consequently will be found by the Propor-

tion of those *Arches*.

Ex. p. 1. Let us suppose $T = t$. Now since

(by *Cor. II.*) 'tis $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$, and $t = T$ (by

Hypothesis) therefore $C : c :: \frac{t^2}{r} : \frac{T^2}{R} :: R : r$.

That is **THEOR.** If the Periodick Times are

equal, the Centripetal Forces are directly as the

Rays. For the Velocities. (Tis $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$

(*Calc.*) (by

(by *Cor. I.*) and $C : c :: R : r$ (in this Case of equal Times.) Therefore $\frac{V^2}{R} : \frac{v^2}{r} :: R : r$. from whence $V : v :: R : r$. *Theor.* The Velocities are directly as the Rays, when the Periodick Times are equal.

2. Let $T^2 : t^2 :: R : r$. Therefore since $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$ (by *Cor. II.*) 'tis plain, that $C = c$. *Theor.* If the Squares of the Periodick Times be directly as the Rays, the Centripetal Forces are equal. For the Velocities. 'Tis $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$ (by *Cor. I.*) and $C = c$ (in this Case;) therefore $\frac{V^2}{R} = \frac{v^2}{r}$, from whence $V^2 : v^2 :: R : r$, and $V : v :: R^{\frac{1}{2}} : r^{\frac{1}{2}}$. *Theor.* The Velocities are directly in the subduplicate Ratio of the Rays, when the Squares of the Periodick Times are as the Rays.

3. Let $T^3 : t^3 :: R^2 : r^2$. Therefore since $C : c :: \frac{t^3}{r} : \frac{T^3}{R}$ (by *Cor. II.*) 'Tis clear, (that $C : c :: r : R$. *Theor.* If the Squares of the Periodick Times be as the Squares of the Rays directly, the Centripetal Forces shall be as the Rays reciprocally. For the Velocities. 'Tis $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$ (by *Cor. I.*) and $C : c :: r : R$ (in this Case)

Case) therefore $\frac{V^2}{R} : \frac{v^2}{r} :: r : R$, from whence

$V^2 = v^2$, and $V = v$. THEOR. If the Squares of the Periodick Times are as the Squares of the Rays, the Velocities are equal.

4. Let $T^2 : t^2 :: R^3 : r^3$. Therefore since

$C : c :: \frac{t^2}{r} : \frac{T^2}{R}$ (by Cor. II.) 'tis plain, that

$C : c :: r^2 : R^2$. THEOR. If the Squares of the Periodick Times be as the Cubes of the Rays, the Centripetal Forces shall be reciprocally as the Squares of the Rays. For the Velocities.

'Tis $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$ (by Cor. I.) and $C : c :: r^2 : R^2$

R^2 (in this present Case) therefore $\frac{V^2}{R} : \frac{v^2}{r} ::$

$r^2 : R^2$, from whence we have $V^2 R = v^2 r$, and

$V : v :: r^{\frac{1}{2}} : R^{\frac{1}{2}}$. THEOR. When the Squares of the Periodick Times are as the Cubes of the Rays, the Velocities are reciprocally in the Subduplicate Ratio of the Rays.

5. Let $T^m : t^m :: R^n : r^n$ universally. There-

fore since $C : c :: \frac{t^2}{r} : \frac{T^2}{R}$ (by Cor. II.) and $T^2 :$

$t^2 :: R^{\frac{2n}{m}} : r^{\frac{2n}{m}}$ (by the Consequence of this

Supposition) therefore $C : c :: r^{\frac{2n}{m}} : R^{\frac{2n}{m}}$, that

L

is,

is, $C : c :: r^{\frac{2n-m}{m}} : R^{\frac{2n-m}{m}}$. For the Velocities. We have $C : c :: \frac{V^2}{R} : \frac{v^2}{r}$ (from Cor. I.)

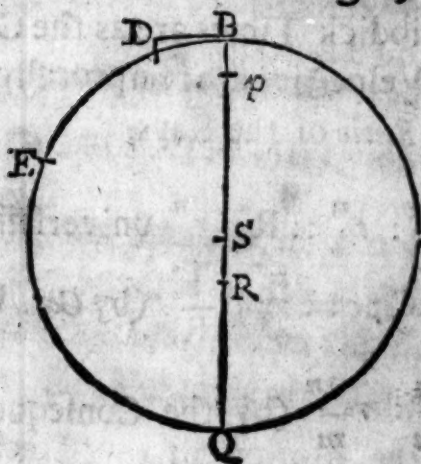
and $C : c :: r^{\frac{2n-m}{m}} : R^{\frac{2n-m}{m}}$, therefore $\frac{V^2}{R} : \frac{v^2}{r} :: r^{\frac{2n-m}{m}} : R^{\frac{2n-m}{m}}$; from whence (by just

Operation with the Exponents) we shall have

$V : v :: r^{\frac{n-m}{m}} : R^{\frac{n-m}{m}}$. So that if the Exponent of the Periodick Times be m , and that of the Rays be $=n$; then the Exponent of the Centripetal Forces shall $=\frac{2n-m}{m}$, and that of the

Velocities shall $=\frac{n-m}{m}$. Q. E. I.

Fig. 5.



PROB.

P R O B. I.

*To determine the Space the Centripetal Force
wou'd carry a Body thro', in the same Time
that it describes any given Arch of a Cir-
cle, in which 'tis suppos'd to move uni-
formly.*

IMagine the Body at the Point B (*FIG. V.*)
to be acted entirely by the Centripetal
Force, and so drawn on by it in the Diameter
BQ towards the Center S. Now, if this Force
be suppos'd to remain still the same, without
any manner of Alteration, and to act upon the
Body by continual repeated Impulses; then the
Body will be carry'd by it after the same man-
ner, and according to the same Law, that it
wou'd be by the Force of Gravity. And there-
fore the Body (thus acted upon by the Centripetal
Force) wou'd describe Spaces in the Duplicate
Ratio of the Times: For that is the *Law of Gra-
vity* (by *Theor. II. Galil. Dial. III.*)

This premis'd; Let the infinitely small
Arch $BD = a$, and the Particle of Time 'tis
describ'd in be put $= t$; also let the given
Arch (intended in the *Problem*, be $BE = A$,
and the Time 'tis describ'd in be $= T$. Then
because the Motion in the Circle is suppos'd
equable, 'tis $t : T :: a : A$, and $t^2 : T^2 :: a^2 :$

A^2 , (from what *Galileus* has demonstrated about equable Motions.) Also (putting the Diameter of the Circle = D) 'tis $a^2 : A^2 :: \frac{a^2}{D} : \frac{A^2}{D}$;

therefore $t^2 : T^2 :: \frac{a^2}{D} : \frac{A^2}{D}$. But it was shewn

before, that the Spaces which the Body (acted by the Centripetal Force) describes, are as the Squares of the Times; and there-

fore those Spaces are as the Quantities $\frac{a^2}{D}$, and

$\frac{A^2}{D}$. But in the Time t , while the little Arch a

is describ'd, the Centripetal Force carries the

Body through the little Space $\frac{a^2}{D}$ (by *Art* II.

Theor. IV.) therefore in the Time T , while the Arch A is describ'd, it wou'd (if the Body were solely acted by it) carry it thro' the Space

$\frac{A^2}{D}$. From whence arises this THEOR. That

in what Time a Body revolving in a Circle describes any given Arch BE , in the same Time, if it were carried by the Centripetal Force in a Rectilineal Course, it wou'd describe a Space

equal to $\frac{BE^2}{BQ}$, that is, to the Square of that

Arch divided by the Diameter. Q. E. I.

COR.

C O R. I.

Bodies describing different Circles with equal Motions; the Spaces (they would be carried thro' by the Centripetal Forces in those Circles, in the same Time) are directly proportional to the Centripetal Forces themselves.

Ex. gr. Let D, d . be the Diameters of Two Circles, having their Center common; a, b . Two infinitely small Arches describ'd in these Circles in the same Particle of Time $= t$. A. B. any Two Arches of a finite Length describ'd, the one in one Circle, and the other in the other, in the same Time T . Then from what was shewn be-

fore $t^2 : T^2 :: \frac{a^2}{D} : \frac{A^2}{D}$, for one Circle; and also

$t^2 : T^2 :: \frac{b^2}{d} : \frac{B^2}{d}$, for the other Circle. There-

fore (by Equality) $\frac{a^2}{D} : \frac{A^2}{D} :: \frac{b^2}{d} : \frac{B^2}{d}$, and $\frac{a^2}{D} :$

$\frac{b^2}{d} :: \frac{A^2}{D} : \frac{B^2}{d}$. But the little Arks a and b be-

ing describ'd in the same Time (by the Hypo-

thesis) the Quantities $\frac{a^2}{D}, \frac{b^2}{d}$ express the Pro-

portion of the Centripetal Forces in those Cir-

cle (by *Art III. Theor. IV.*) Also the Quantities

$\frac{A^2}{D}, \frac{B^2}{d}$ are the Spaces the Bodies are carry'd

thro' by the Centripetal Forces in those Circles in the Time T (by the Supposition.) Therefore the Consequence is clear.

C O R. II.

Supposing all as before; the Spaces that the Bodies wou'd be carry'd thro' by the Centripetal Forces, in unequal Times, are in the *Ratio* compounded of the Squares of the Times, and the Centripetal Forces. All the other Symbols standing, let the Arch B be describ'd in the Time θ ; the infinitely small Arks a and b being supposed to be describ'd in the same Particle of Time t , as before. Then $t^2 : T^2 :: \frac{a^2}{D} : \frac{A^2}{D}$, for one Circle, and $t^2 : \theta^2 :: \frac{b^2}{d} : \frac{B^2}{d}$, for the other Circle. Therefore (equalling these Two Values of t^2 , and making the Analogies from thence) we have $\frac{A^2}{D} : \frac{B^2}{d} :: T^2 \times \frac{a^2}{D} : \theta^2 \times \frac{b^2}{d}$. But the Two former Terms of the Analogy are the Spaces the Bodies wou'd be carry'd thro' by the Centripetal Forces in the Time T and θ ; and the Two latter Terms are compounded of the Squares of those Times, and the Centripetal Forces in the Two Circles. Therefore, &c.

COR.

C O R. II.

If the Symbols A and B, instead of Arches, denote the whole Circumferences of the Circles, and consequently T and θ be the whole Periodick Times of the Description of those Circumferences; then 'tis evident, that the Quantities

$\frac{A^2}{D}$, $\frac{B^2}{d}$, will be to one another as $D : d$, that is,

the Spaces (the Centripetal Forces wou'd carry the Bodies thro' in the Times that the Circumferences are describ'd) wou'd be in Proportion as the Diameters, or the Circumferences.

For $A : B :: D : d$ (that is, the Peripheries are

as the Diameters) and so $\frac{A^2}{D} : \frac{B^2}{d} :: \frac{D^2}{D} : \frac{d^2}{d} ::$

$D : d$. And there's no need of any Proof that this must be true universally, whatever be the Ratio of the Periodick Times, or in what unequal Times soever the Circumferences of the Circles are imagined to be describ'd by the

Two Bodies. However, if there were, the young Student for his Exercise might easily clear it, from the general Analogy in the last Corol-

lary. For there 'twas shewn, that $\frac{A^2}{D} : \frac{B^2}{d} ::$

$T^2 \times \frac{a^2}{D} : \theta^2 \times \frac{b^2}{d}$: Now, the Quantities $\frac{a^2}{D}$, $\frac{b^2}{d}$

expressing the Centripetal Forces in these Circles, let us use the Symbols C. c. for them, and

then the Analogy stands thus, *viz.* $\frac{A^2}{D} : \frac{B^2}{d} :: T^2 \times C : \theta^2 \times c$. Now, here let the Periodick Times be in what *Ratio* of the Rays we please, we shall still find $\frac{A^2}{D} : \frac{B^2}{d} :: D : d$, *Ex. gr.* Let the Periodick Times be equal, that is, $T = \theta$, then will $C : c :: D : d$ (by what was shewn above at the *Corollaries* of *Theor.* IV.) and consequently $\frac{A^2}{D} : \frac{B^2}{d} :: D : d$. Again, let $T^2 : \theta^2 :: D : d$, then will $C = c$, and so $\frac{A^2}{D} : \frac{B^2}{d} :: D : d$. Let $T^2 : \theta^2 :: D^3 : d^3$, then will $C : c :: d^2 : D^2$, and consequently $\frac{A^2}{D} : \frac{B^2}{d} :: D^3 \times d : d^3 \times D^2 :: D : d$. And so proportionably in any other Supposition that may be made.

C O R. IV.

From what is found in this *Problem*, we may easily compare Centripetal Forces with any other known Force, such as that of Gravity (*Ex. gr.*) which shall be the Business of the following *Problem*.

PROB. I. Now, the Centripetal Force in this Case, let us use the Symbols C for it, and c for it, then

P R O B. II.

To find the Proportion of the Centripetal Force to that of Gravity.

U Sing the same Symbols as in *Prob. I.* suppose in the Particle of Time t , while the little Ark $BD=a$ is describ'd, (*FIG. V.*) the Body by the Force of its Gravity describ'd the little Space BP , which put $=p$. Then in the Time T , while the Arch $BE=A$ is describ'd, the Body will have descended through a Space $= \frac{T^2 \times p}{t^2}$ (for $t^2 : T^2 :: p :$ to the Space describ'd in the Time T , by the *Law of Gravity*) $= BR$

(*Ex.gr.*) But $\frac{T^2 \times p}{t^2} = \frac{A^2 \times p}{a^2}$ (since $t^2 : T^2 :: a^2 : A^2$, because of the equable Motion in the Circle)

So that $BR = \frac{A^2 \times p}{a^2} =$ the Space describ'd by the Gravity in the Time T . But in the same Time, the Centripetal Force wou'd carry the

Body thro' a Space $= \frac{A^2}{D}$ (by what was shewn at *Prob. I.*) Therefore these Spaces are to one

another as $\frac{A^2 \times p}{a^2} : \frac{A^2}{D}$, that is, as $\frac{p}{a^2} : \frac{1}{D}$, or as

$D \times p : a^2$. And consequently the Force of Gravity, and the Centripetal Force, making the Body

Body to describe these Spaces in the same Time, are one to another in the same *Ratio*.
Q. E. I.

But because the Terms p and a are infinitely small, therefore to express this Proportion in finite Quantities, we may proceed thus. Let it be $t^2 : T^2 :: p : k$; then k is the Space describ'd by the descending Body in the Time T . But (from the Uniform Motion in the Circle) $t^2 : T^2 :: a^2 : A^2$, therefore (by Equality) $a^2 : A^2 :: p : k$, and $a^2 : p :: A^2 : k$, and consequently instead of the Terms p and a^2 , putting in k and A^2 , which are proportional to them, the Proportion will be as $D \times k : A^2$. So that we have this *Theorem*, The Force of Gravity is to the Centripetal Force (of a Body revolving uniformly in a Circle) as the Rectangle under the Diameter of the Circle, and the Line describ'd by a descending Body in any given Time, to the Square of the Arch describ'd by the revolving Body in the same Time. Q. E. F.

C O R.

When $D \times k = A^2$, then those Two Forces are equal; that is, when $k = \frac{A^2}{D}$, or the Space a Body describes by the Force of Gravity, is equal to that it would be carried thro' by the Centripetal Force in the same Time.

THEOR.

T H E O R. V.

A Body revolving in the Circumference of a Circle, the Centripetal and Centrifugal Forces are equal to one another.

FOR while the little Arch (*FIG. V.*) BD (*Ex. gr.*) is describ'd, the Centripetal Force has carried the Body towards the Center by the Space CD. And if the Centripetal Force were away, the Body, instead of describing the Arch BD, wou'd go on in the Tangent, and describe in the same Time the *Lineola* BC (which by *Lemma I.* is = the little Arch BD.) In which Case it wou'd be carried off from the Curve of the Circle, by the Distance of the same *Lineola* CD; and this is the Effect of the Centrifugal Force. Therefore the Centripetal and Centrifugal Forces producing an Effect of the same Magnitude in the same Time, are equal to one another. Q. E. D.

C O R. I.

All the *Theorems* relating to Centripetal Forces, which were deduced in the *Corollaries* of *Theor. IV.* are applicable to Centrifugal Forces also.

COR.

C O R. II.

And, according to the Tenour of the last Problem, the Centrifugal Force shall be to that of Gravity, as $A^2 : D \times k$; and consequently equal to it, when $A^2 = D \times k$. But to explain here a little what I intend by the Centrifugal Force being equal to that of Gravity, let us conceive a Body revolving in a Circle to be held by a Thread, produced out of the Center of the Circle it describes, so that by the Means of this Thread it keeps always exactly in the Circumference of the Circle. 'Tis certain, that as the Body revolves with a greater or less Degree of Velocity, so the Thread that holds it will be stretch'd more or less violently. For the Endeavour to recede from the Center of its Motion will be greater or less, as the Velocity with which it moves is either greater or less; and that greater or less Endeavour of receding will produce a greater or less Tension of the Thread. This stretching of the Thread by the Body, as it moves in this manner circularly, is the Effect of the Centrifugal Force, as that Tension of it by the meer Weight of the Body, where it is freely suspended by the same, without the Intervention of any sort of Motion or Impediment, is the compleat Effect of the Force of Gravity.

Gravity. And as the Degrees of Velocity (with which a Body may move circularly) are infinite, and consequently there may be as many Degrees of Tension of the Thread that detains it in its Path in that Motion; so when such a Degree of Velocity is pitch'd upon, that the Tension of the Thread arising from thence, is equal to that which is the Result of the free Suspension; we say then, that the Centrifugal Force is equal to that of Gravity, because they both produce the same Effects upon the Thread; or whatever Proportion the Tensions bear to one another, we may say, the Forces (the Causes of them) bear the same. Now, to enquire a little into this Matter. The revolving Body moving uniformly in the Circle, we will consider the Velocity of that Motion, as some one or other of those Velocities, which would be acquir'd by it in a free perpendicular Descent, *Ex. gr.* in the Diameter of the Circle. Now, whatever Space the Body describes by the Force of Gravity in its Descent, if it afterwards moves equably with the greatest Velocity gotten by that Descent, it's a known *Theorem*, That the Space describ'd by the uniform Motion shall be double that describ'd by the uniformly Accelerate, in the same Time. Let the Body therefore in the Time T descend thro' the Space k , and if with the Velocity gotten by this

this Descent it afterwards moves equably in the Circle, and does in the same Time T describe the Arch A ; then shall $A=2k$. Now, let $k=mD$ universally, where m denotes either an Integer, or a Fraction. Then is $A=2mD$, and $\frac{A}{D}$

(the Space the Centrifugal Force would carry the Body thro' in the Time T) $=4mmD$. Therefore the Space described by the Centrifugal Force, is to that described by the Gravity in the same Time T , as $4mmD:mD$, that is, as $4m:1$, and consequently these Forces are in the same Ratio. Hence in particular Cases the Proportion of the Forces is easily known. *Ex. gr.* If $m=\frac{1}{4}$, then $4m=1$, and so the Centrifugal Force in that Case is equal to the Gravity. If $m=\frac{1}{2}$, then $4m=2$, and so the Proportion of the Forces is as 2. 1. If $m=\frac{3}{4}$, then the Proportion is as 3. 1. If $m=1$, then 'tis as 4. 1. and in like manner for any other sort of Supposition. The Sense of all which may be express'd in this *Theorem*, That if a Body revolves in a Circle, with a Velocity equal to that acquir'd by a Descent thro' a Space, that is to the Diameter of the Circle, as m to Unity; that then the Centrifugal Force shall be to that of Gravity, as $4m$ to Unity.

SCHOL.

S C H O L. I.

Having hinted a certain *Mechanick Theorem* before concerning the Proportion of the Spaces describ'd by an equable and an uniformly accelerated Motion, in the same Time, I think it not improper to subjoin a short Illustration of it. Let the Quantity x express Time, y Velocity, the greatest Time d , the greatest Velocity b ; the Two former Quantities being flowing ones, and the latter standing. From the Nature of the uniformly accelerate Motion, the Velocities are proportional to the Times. Therefore $x : y :: d : b$, and $x = \frac{yd}{b}$, and $\dot{x} = \frac{\dot{y}d}{b}$.

Also in all Motions whatsoever, the Spaces described are as the Rectangles under the Velocities and the Times. Therefore the Fluxion of the Space for the accelerate Motion, is $= y\dot{x}$, and that for the equable Motion is $= b\dot{x}$. But the Fluent of $b\dot{x}$ is $= bx = bd$, and the Fluent of $y\dot{x}$ ($= \frac{yy\dot{y}d}{b}$) is $= \frac{yyd}{2b} = \frac{bbd}{2b} = \frac{bd}{2}$. There-

fore the Space describ'd by the equable Motion in the Time d with the standing Velocity b , is double the Space describ'd by the uniformly Accelerate, in the same Time d , with a flowing Velocity, the greatest of which is equal to b (the Velocity of the uniform Motion.) And
From

from hence to determine the Proportion of the Spaces, describ'd by an equable Motion with the Velocity b , and in the Time d , *Ex. gr.* and an uniformly accelerated one in the Time nd , where n denotes any whole or broken Number. It is (from the Nature of uniformly accelerated Motion, in which the Spaces are as the Squares of the Times) $d^2 : n^2 d^2 :: \frac{bd}{2} : \frac{n^2 d^2 b}{2d^2}$
 $= \frac{mbd}{2}$ = the Space describ'd by the accelerate Motion in the Time nd ; the Space describ'd by it in the Time d being $= \frac{bd}{2}$, by what was shewn before. So that the Proportion required is that of $bd : \frac{mbd}{2}$, that is, of $1 : \frac{m}{2}$, or of $2 : m$ universally; which may be exemplified in particular Cases at Liberty.

SCHOL. II.

Now, to apply this to the former Comparison between the Centrifugal Force and that of Gravity. Let the Arch A be describ'd by the Body revolving in a Circle with an uniform Motion, in the Time d , and the Line k describ'd by its Descent with an uniformly accelerate Motion in the Time nd ; according to the latter part of the foregoing *Scholium*. Then
it

is universally $A : k :: 2 : mn$, and $A = \frac{2k}{mn}$, from

whence $\frac{A^2}{D}$ (the Space describ'd by the Centri-

fugal Force in the Time d) is $= \frac{4kk}{Dn^2}$. Suppose

(as before at *Cor. II.*) $k = md$, then $\frac{4kk}{Dn^2} = \frac{4mmDD}{Dn^2}$

$= \frac{4mmD}{n^2}$; and consequently the Proportion of

the Spaces describ'd by the Centrifugal Force

in the Time d , and by the Gravity in the Time

nd , is as $\frac{4mmD}{n^2} : mD$, that is as $\frac{4m}{n^2} : 1$, or as

$4m : n^2$. Of which more general Proportion,

that at *Cor. II.* is but a particular Case, *viz.*

when the Times are equal, or when n is equal

to Unity.

Ratio of their Homologous Sides.

M

LEMMA.

Approach of the Points B, C to A , the

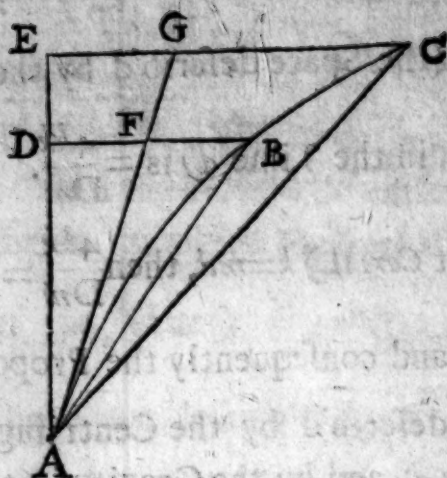
Ratio of the Tangents AC, AB , the Chords $AC,$

AB , and the Arches AC, AB is a Ratio of E-

quality (by Lemma I.) Therefore the Curvi-

linear Triangles ADE, AEC are at last equal

Fig. 6.



L E M M A.

If the Right Line *AE* (FIG. VI.) and the Curve *AC*, given in Position, cut one another in the given Angle *A*; and if the Right Lines *DB*, *EC* be ordinately apply'd to the Line *AE* (in another given Angle) meeting the Curve in the Points in *B*, *C*; Then supposing the Points *B*, *C*, to come to the Points *A*, the Curvilineal Triangles *ADB*, *AEC*, shall be at last in the duplicate Ratio of their Homologous Sides.

FOR drawing the Tangent *AFG*; upon the Approach of the Points *B*, *C*, to *A*, the Ratio of the Tangents *AG*, *AF*, the Chords *AC*, *AB*, and the Arches *AC*, *AB* is a Ratio of Equality (by Lemma I.) Therefore the Curvilineal Triangles *ADB*, *AEC* are at last coincident

dent with the Rectilineal Triangles ADB, AEG; but these being similar (by the *Hypotbesis*) are in a duplicate *Ratio* of their Homologous Sides (by the *Elements*;) therefore the Curvilineal ones are so too.

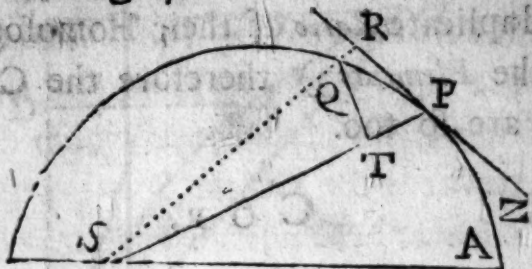
C O R.

It is evident, that the Spaces which a Body (urged by any regular Force) describes, are in the Beginning of the Motion as the Squares of the Times: For if the Times be express'd by the Lines AD, AE, and the Velocities in those Times by BD, EC, then (from *Mechanicks*) the Spaces shall be as the *Area's* ADB, AEC; that is, in the Beginning of the Motion, as the Squares of the Times AD, AE.

M 2

THEOR.

Fig. 7.



THEOR. VI.

If a Body revolves (FIG. VII.) about a certain Center *S*, and describes a Curve *APQ*; then if *ZPR* touches the Curve in *P*, and from any other Points *Q* be drawn *QR* parallel to *SP*, and *QT* perpendicular to the same; the Centripetal Force shall be reciprocally as the solid $\frac{SP^3 \times QT^3}{QR}$; taking the Value of it, not where the Points *P* and *Q* are at any finite Distance, but at last when they come together.

PUT *C* for Centripetal Force, and *T* for Time. Now, in the indefinitely small Figure *QRPT*, the *Lineola* *QR*, being the genuine Effect of the Centripetal Force, will be in a gi-

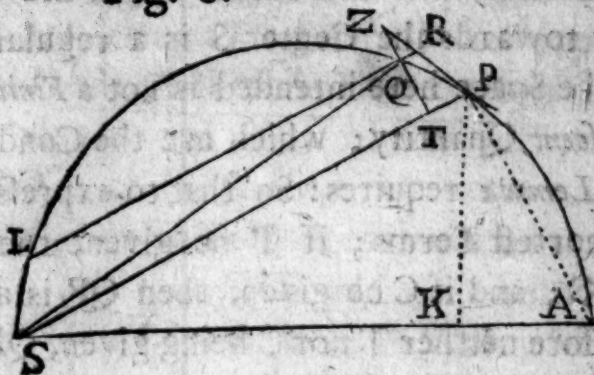
ven Time proportional to the Force, (by the Second *General Law of Motion*.) But if the Force it self were given, the *Lineola* QR would be as the Square of the Time (by the foregoing *Lemma*) for the Force which draws the Body towards the Center S is a regular one, and the Space here intended is not a *Finite*, but a *Nascent* Quantity; which are the Conditions that *Lemma* requires: So that to express it in the shortest Terms; if T be given, then QR is as C; and if C be given, then QR is as T²; therefore neither T nor C being given, QR is in the *Ratio* compounded of both, that is, QR is as C × T², and consequently C is as $\frac{QR}{T^2}$. But the Time is as the *Area* (by *Theor. I.*) that is, T is as $\frac{1}{2} SP \times QT$ (= the *Area* SPQ) and $\frac{1}{2} SP \times QT$ is as $\frac{QR}{SP \times QT}$; so that C is as $\frac{QR}{SP \times QT}$ directly, that is as $\frac{SP \times QT^3}{QR}$ Reciprocally. Q.E.D.

C O R,

Hence if any Curve be given, and within it a Point, to which (as a Center) the Centripetal Force is directed; we may find the Law of the Centripetal Force, which makes the Body to revolve in the Periphery of that Curve. In order to this, we are to compute the Value of this

solid $\frac{SP^q \times QT^q}{QR}$, according to the Nature of the Curve: Some Examples of which follow.

Fig. 8.



P R O B. III.

A Body revolving in the Circumference of a Circle, 'tis requir'd to find the Law of the Centripetal Force tending to a Given Point in the Circumference.

LET P (*FIG. VII.*) be the Place of the Body in the Periphery SQPA, Q the next Place into which it moves, and S the Center to which the Forces are directed. Let PK be perpendicular from P to the Diameter SA, QT perpendicular from Q to SP, LQR parallel to SP, RP a Tangent at P meeting QT, produced in Z. In the First place here, the Triangles ZQR, ZTP, SPA are similar. That the Triangles ZQR, ZTP are similar is plain, because

QR

QR is parallel to the Line SP by the Construction. Again, that the Triangles ZTP, SPA are similar, is evident, for the Angle SPA is a Right one from the Nature of the Circle, and the Angle ZTP is a Right one by the Construction; also ZP being a Tangent, and SP a Chord, cutting the Circle at P, the Point of Contact, the Angle ZPT is = Angle SAP (standing upon the Arch SQP) from the *Elements*. Therefore the Angle TZP = Angle PSA, and consequently the Triangles are similar. From whence $ZP^q : ZT^q :: SA^q : SP^q$, and $ZP^q : ZT^q :: RP^q : QT^q$ (because of the proportional Section of the Sides ZP and ZT by the Parallel QR;) therefore $RP^q : QT^q :: SA^q : SP^q$. But RP being a Tangent, and RQL a Secant, cutting it in R, and the Circle in Q, therefore (from the *Elements*) $RP^q =$ the Rectangle $QR \times RL$, therefore the Analogy stands thus $QR \times RL : QT^q :: SA^q : SP^q$; from whence

$$QT^q = \frac{QR \times RL \times SP^q}{SA^q}; \text{ And because when the}$$

Points Q and P come together, the Line RL becomes = SP, therefore putting SP for RL, we

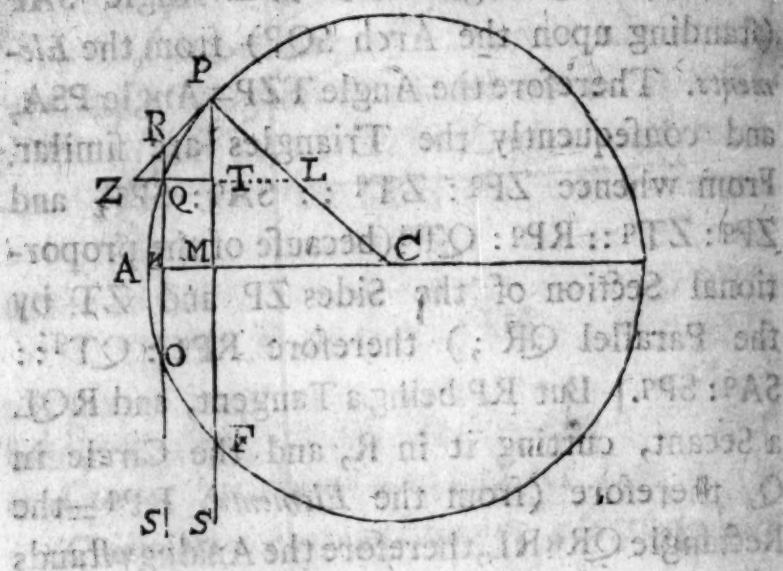
$$\text{have } QT^q = \frac{QR \times SP^3}{SA^q}. \text{ Now (by Theor. VI.) the}$$

Centripetal Force is reciprocally as $\frac{SP^q \times QT^q}{QR}$,

and we have found the Value of QT^q in the proper Terms of the Curve, therefore multi-

plying this Value of QT^q by $\frac{SP^q}{QR}$, we have $\frac{SP^q}{SA^q}$, which is Reciprocally as the Centripetal Force in this Case; that is, because SA, the Diameter, is a given Quantity, the Centripetal Force reciprocally as SP^q , that is the Square-Cube of the Distance from the Center S. Q. E. I.

Fig. 9.



P R O B. IV.

A Body revolving in the Circle PQA (Fig. 9.) 'tis requir'd to find the Law of the Centripetal Force tending to a Point S so far distant, that all the Lines, as PS, RS, &c. drawn to that Point, may be esteem'd as parallel.

FROM the Center C draw CA, cutting the Parallels PS, RS, at Right Angles; join the Points C, P, and produce ZT to cut CP in L.

L. Here ZP is a Tangent, as before, and the Line ZTL at Right Angles to the Parallels PS and RS. Now, the Triangles ZRQ, ZPT, CPM are similar. For ZT cutting the Parallels PS, RS at Right Angles, 'tis plain, that the Triangles ZRQ, ZPT are similar. Also, because the Angle ZPC is a Right one, and the Line PT a Perpendicular from the Right Angle to the Base ZL; therefore the Triangle ZPT is similar to the Triangle PTL; and since the Triangle PTL is similar to PMC; therefore ZPT and PMC are similar. Therefore $ZP^q : ZT^q :: CP^q : PM^q$; but $ZP^q : ZT^q :: RP^q : QT^q$ (from the proportional *Section* of the Sides ZP and ZT by the Parallel RQ;) therefore $RP^q : QT^q :: CP^q : PM^q$. But (from the Nature of the Circle RP being a Tangent, and RQ a Secant, cutting it in the Point R, and the Circle in O) $RP^q = OR \times QR$; from whence

$$QT^q = \frac{RP^q \times PM^q}{CP^q} \text{ is } = \frac{OR \times QR \times PM^q}{CP^q}. \text{ But}$$

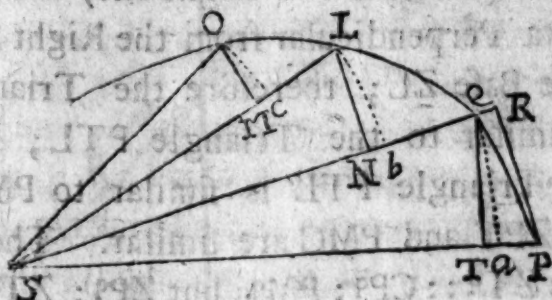
when the Points P and Q meet together, OR becomes = PF = 2PM; therefore putting 2PM for OR, we have $QT^q = \frac{2PM \times QR}{CP^q}$; and so the

general Formula $\frac{SP^q \times QT^q}{QR}$ is equal $\frac{2PM \times SP^q}{CP^q}$.

Now, the Point S being at an infinite Distance, SP is a standing Quantity, and so is CP the Radius; so that dividing by the standing Quantity

$\frac{1}{2} \frac{SP^2}{CP^2}$, the Expression amounts to PM^2 . Therefore the Centripetal Force is reciprocally as the Cube of the Ordinate PM . Q. E. I.

Fig. 10.



P R O B. V.

A Body revolving in the Logarithmick or Proportional Spiral, 'tis requir'd to find the Law of the Centripetal Force tending to the Center of the Spiral.

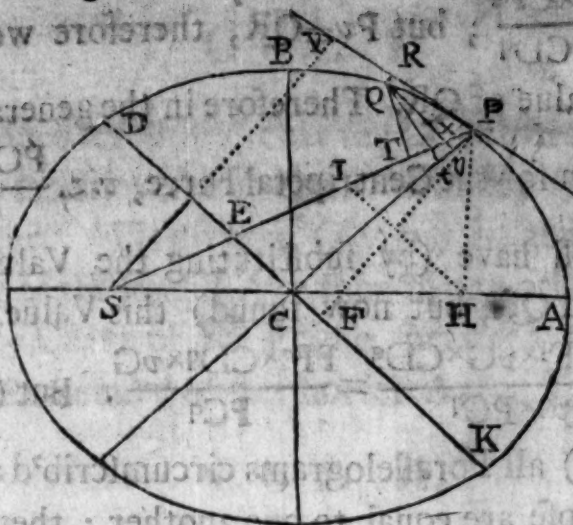
THE Nature of this Curve (FIG. X.) is such, that all the Angles, as SPQ , SQL , SLO , &c. (which the Rays make with the Curve or Tangents at those Points) are still of the same Magnitude. From whence it follows, that taking all the indefinitely small Angles about the Center S continually equal, that is $PSQ = QSL = LSO$, &c. the Triangles PSQ , QSL , LSO , &c.

are

are similar, and therefore $PS : QS :: QS : SL ::$
 $SL : SO$, &c. Consequently the Rays PS , QS ,
 SL , &c. are in the continual Geometrick Pro-
portion. Therefore since $SP : SQ :: SQ : SL$,
'tis also $SP : SP - SQ :: SQ : SQ - SL$, that is (de-
scribing the indefinitely little Arches Qa , Lb ,
 Oc , upon the Center S) we have $SP : Pa ::$
 $SQ : Qb$ or (which amounts to the same, let-
ting fall the Perpendiculars QT , LN , OM) 'tis
 $SP : PT :: SQ : QN$. So that the *Lineole* PT ,
and QN , are always as their respective Rays SP
and QS . Lastly, because the indefinitely small
Triangles PQT , QLN , &c. are always similar
(for the Angles at P and Q are equal from the
Nature of the Curve, and those at T and N are
Right ones) therefore $PT : QT :: QN : LN$,
and so PT and QN are always proportional to
 QT and LN . But before 'twas shewn, that
 PT and QN were always as their respective
Rays SP and QS ; therefore also the *Lineole*
 QT and LN are ever as those Rays SP and QS .
I thought this little Explication of the most
common Properties of the Curve, necessary to
be premis'd, in order to the understanding our
Great Author's Process in the Invention of the
Law of the Centripetal Force. Now, let the
indefinitely small Angle PSQ be given or sup-
posed always of the same Bigness; then in
the Triangle SPR , since the Angle at S is al-
ways the same, and that at P also ever the same
(from

(from the Nature of the Curve) therefore the Angle at R, viz. SRP, is ever the same. So in the Triangle SQT, since the Angle at T is ever a Right one, the Angle SQT shall always be the same. From hence therefore in the indefinitely small Figure RQTP, the Angles at P, T and R are standing, and since the Angle SQT was shewn to be so, then RQT, the Complement of SQT to Two Right Angles, shall be of a standing Magnitude too. Therefore all the Angles of the Figure RQTP, are of a standing Magnitude, and consequently the Species of the Figure it self is standing, or always given. Therefore the *Ratio* of the Sides QT, QR, viz. $\frac{QT}{QR}$, is a standing Quantity, and so $\frac{QT}{QR}$ may be always as 1. Consequently if $\frac{QT}{QR}$ be as 1, then (multiplying both by QT) $\frac{QT^2}{QR}$ shall be always as QT. But (as was shewn before) from the Nature of the Curve, QT is as SP; therefore also $\frac{QT^2}{QR}$ shall be as SP, therefore $\frac{QT^2 \times SP}{QR}$ (which is reciprocally as the Centripetal Force) shall be as SP³. Therefore the Centripetal Force in this Curve is Reciprocally as the Cube of the Distance from the Center S. Q. E. I.

Fig. 11.



P R O B. VI.

A Body revolving in an Ellipse, 'tis requir'd to find the Law of the Centripetal Force of a Body tending to the Center of that Ellipse.

LET CA, CB, (FIG. XI.) be the *Semiaxes* of the Ellipse; GP, DK conjugate *Diameters*; PF, Qt *Perpendiculars* to those *Diameters*; Qv an *ordinate* *Applicate* to the *Diameter* GP; and complete the *Parallelogram* QvRP: The *Triangles* Qtv, PCF are *similar*, for the *Angles* at t and F are *Right ones* by the *Hypothesis*; but Qv being *parallel* to CF the *Angle* Qvt = FCP; therefore $Qt^q : Qv^q :: PF^q : PC^q$, and $Qt^q = \frac{Qv^q \times PF^q}{PC^q}$. But (from *Conick Sect.*) $Qv^q : Pv \times vG :: CD^q : CP^q$; therefore Pv

$= \frac{Qv^q \times CP^q}{vG \times CD^q}$; but $Pv = QR$, therefore we have

this Value of QR . Therefore in the general Expression for the Centripetal Force, viz. $\frac{PC^q \times Qv^q}{QR}$

we shall have (by substituting the Values of Qv^q and QR , but now found) this Value, viz.

$$\frac{Qv^q \times PF^q \times vG \times CD^q}{Qv^q \times PC^q} = \frac{PF^q \times CD^q \times vG}{PC^q}. \text{ But (from}$$

Conicks) all Parallelograms circumscrib'd about an Ellipse are equal to one another; therefore $CD \times PF = BC \times CA$, and $CD^q \times PF^q = BC^q \times CA^q$, and substituting $BC^q \times CA^q$ in the Room of $CD^q \times PF^q$, the Expression comes to $\frac{BC^q \times CA^q \times vG}{PC^q}$.

But when the Points P and Q come together, then vG is $= 2PC$, so that then the Expression amounts to this, viz. $\frac{BC^q \times CA^q \times 2PC}{PC^q}$. But

$2BC^q \times CA^q$ is a standing Quantity, therefore the Expression comes to $\frac{PC}{PC^q}$, i. e. $\frac{1}{PC}$; so that

the Centripetal Force is reciprocally as $\frac{1}{PC}$, i. e. directly as PC , the Distance from the Center. Q. E. I.

Cor.

C O R. I.

Hence it follows, that the Centripetal Force is greatest at the greatest Distance from the Center. 'Tis greater (*Ex.gr.*) in the Extremity of the longer Axe, than in any other Point in the Curve of the Ellipse, between that and the Extremity of the shorter Axe.

C O R. II.

At equal Distances from either of the principal Vertices, the Centripetal Forces are equal. For the Diameters or Distances from the Center (from the Nature of the *Ellipse*) are so in this Case.

C O R. III.

Any Two equal Diameters being taken thus, one of one Side the principal Vertex, and the other of the other Side; the Conjugates of these equal Diameters being equal also, it's evident, that the Sum of the Centripetal Forces in the Extremities of one of these Diameters, and its Conjugate, shall be equal to the Sum of the Forces in the Extremities of the other Diameter, and its Conjugate. *Ex.gr.* Let A be any Diameter, *a* its Conjugate; B a Diameter taken on the other Side at an equal Distance from the principal Vertex, and consequently equal

to

C O R. V.

In Ellipses that have the longer Axe common, drawing any Diameter, as AC (FIG. XII.) cutting the Curves in C and *d*, the Centripetal Force in *d* shall be to the Centripetal Force in C, as $A d : A C$. Let the Force in B be put $= F$, Force in *d* $= f$, Force in C $= \phi$. Then the Ratio of the Force in *d* to the Force in C is compounded of the *Rationes* of the Force in *d* to that in B, and of the Force in B to that in C.

That is, $\frac{f}{\phi} = \frac{f}{F} \times \frac{F}{\phi}$. But $\frac{f}{F} = \frac{A d}{A B}$, and $\frac{F}{\phi} =$

$\frac{A B}{A C}$ (from the Law of the Centripetal Force

in each Ellipse) therefore $\frac{f}{\phi} = \frac{A d}{A B} \times \frac{A B}{A C} = \frac{A d}{A C}$;

so that $f : \phi :: A d : A C$. In Two Ellipses therefore that have the same Transverse Axe, the Centripetal Force (which tends to their common Center) is greatest in the exterior Ellipse.

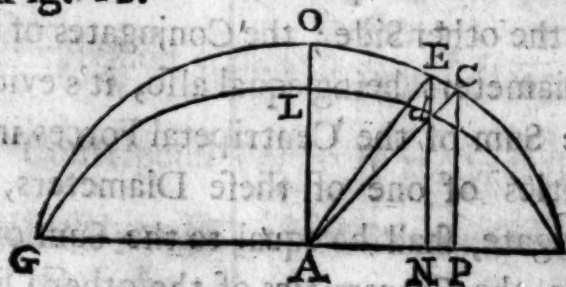
And in the Extremities of the shorter Axes, viz. in the Points L, O, the Centripetal Forces, will be as those Axes, viz. as $A L : A O$; or (which is all one) drawing any Ordinate, as E *d* N cutting the Ellipses in E and *d*, the Forces in the Two Ellipses will be as $d N : E N$; for 'tis $A L : A O :: d N : E N$, from the Nature of the Ellipse.

to A ; and b its Conjugate, which is therefore equal to a . Let F = the Centripetal Force in A , f = Force in a , ϕ = Force in B , and π = Force in b . Now I say, that because $A + a = B + b$, therefore $F + f = \phi + \pi$. For (by the Law of the Centripetal Force found in the last *Problem*.) 'Tis $F : f :: A : a$, and $\phi : \pi :: B : b$, from whence since $A + a = B + b$, also $F + f$ shall $= \phi + \pi$, that is, the Sums of the Centripetal Forces in the Extremities of each Pair of these Conjugate Diameters shall be equal.

C O R. IV.

Therefore also $F - \phi$, or $\phi - F$ is $= \pi - f$, or $f - \pi$; that is, the Difference of the Forces in the Extremities of the Diameters A and B , is equal to the Difference of the Forces in the Extremities of their Conjugates a and b .

Fig. 12.



COR.

C O R. O V.

In Ellipses that have the longer Axe common, drawing any Diameter, as AC (FIG. XII.) cutting the Curves in C and d , the Centripetal Force in d shall be to the Centripetal Force in C, as $Ad : AC$. Let the Force in B be put $= F$, Force in $d = f$, Force in C $= \phi$. Then the Ratio of the Force in d to the Force in C is compounded of the *Rationes* of the Force in d to that in B, and of the Force in B to that in C.

That is, $\frac{f}{\phi} = \frac{f}{F} \times \frac{F}{\phi}$. But $\frac{f}{F} = \frac{Ad}{AB}$, and $\frac{F}{\phi} = \frac{AB}{AC}$ (from the Law of the Centripetal Force in each Ellipse) therefore $\frac{f}{\phi} = \frac{Ad}{AB} \times \frac{AB}{AC} = \frac{Ad}{AC}$; so that $f : \phi :: Ad : AC$. In Two Ellipses therefore that have the same Transverse Axe, the Centripetal Force (which tends to their common Center) is greatest in the exterior Ellipse. And in the Extremities of the shorter Axes, viz. in the Points L, O, the Centripetal Forces, will be as those Axes, viz. as $AL : AO$; or (which is all one) drawing any Ordinate, as EdN cutting the Ellipses in E and d , the Forces in the Two Ellipses will be as $dN : EN$; for 'tis $AL : AO :: dN : EN$, from the Nature of the Ellipse.

C O R. VI.

Supposing all as in the last *Corollary* (with its *Fig.*) from the Point C in the exterior Ellipse, draw the Ordinate CP. Then shall the *Ratio* of the Forces in the Points O and L, be to the *Ratio* of the Forces in the Points C and d, as $\sqrt{GN \times NB} : \sqrt{GP \times PB}$. For the *Ratio* of the Forces in the Points O and L is $= \frac{AO}{AL} = \frac{EN}{dN}$, by what was shewn at the latter End of the preceding *Corollary*. Also the *Ratio* of the Forces in C and d, is $= \frac{AC}{Ad}$, by what was shewn in the former Part of the foregoing *Corollary*. But because dN and CP are perpendicular to GB (by the *Hypothesis*) therefore from the similar Triangles ACP, AdN, we have $\frac{AC}{Ad} = \frac{CP}{dN}$, from whence $\frac{CP}{dN}$ is = the *Ratio* of the Forces in the Points C and d. Therefore the *Ratio* of the Forces in the Points O and L, is to the *Ratio* of the Forces in the Points C and d, as $\frac{EN}{dN} : \frac{CP}{dN}$. But $\frac{EN}{dN} : \frac{CP}{dN} :: EN : CP$, and $EN : CP :: GN \times NB : GP \times PB$ (from the Nature of the Ellipse)

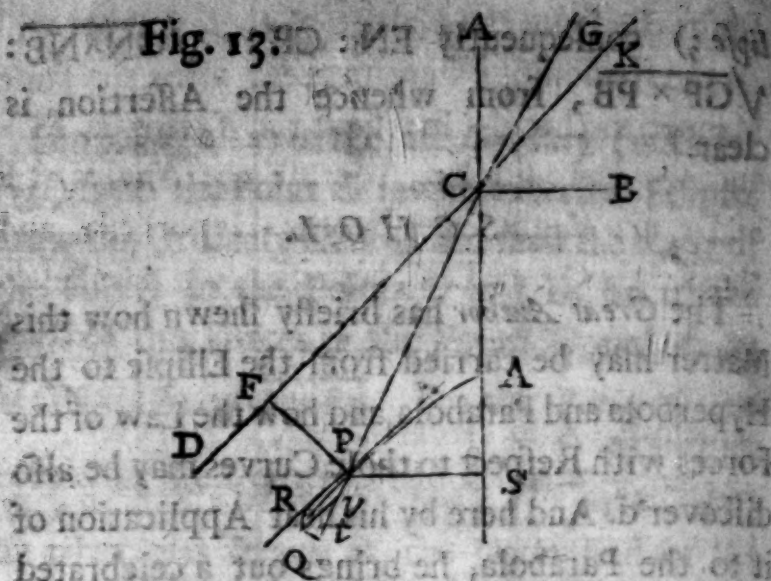
lipse;) consequently $EN : CP :: \sqrt{GN \times NB} : \sqrt{GP \times PB}$, from whence the Assertion is clear.

SCHOL.

The *Great Author* has briefly shewn how this Matter may be carried from the Ellipse to the Hyperbola and Parabola, and how the Law of the Forces with Respect to those Curves may be also discover'd. And here by his neat Application of it to the Parabola, he brings out a celebrated *Theorem of Galileus*. But because the young Beginners in these Things may possibly desire a more particular Explication of it, upon their Account I shall make the Application to these Two Curves distinctly.

This is evident here in the first place, that the Force tending to the Center, is not properly a Centrifugal Force, for the Body will not move and rather than that it will move in a straight line. Let (F, G, X, H) be the Center of the Hyperbola, CP a Tangent, Q an Ordinate to CP , Q and PT perpendiculars to the Diameters CF and CD respectively, and RQ parallel to PT . In a Word, all the Lines drawn here are Analogous to those in the Scheme for the corresponding Case in the El-

Fig. 13



P. R. O. B. VII.

If a Body moves in the Curve of an Hyperbola, 'tis requir'd to find the Law of the Force tending to the Center of the Hyperbola.

TIS evident here in the First place, that the Force tending to the Center, is most properly a Centrifugal Force, for the Body still goes farther and farther from that Point to which the Forces are directed. Let (*FIG. XIII.*) *C* be the Center of the Hyperbola, *CP* a Diameter, *CD* its Conjugate, *RP* a Tangent at *P*, *Qv* an Ordinate to *CP*, *Qz* and *PF* Perpendiculars to the Diameters *CP* and *CD* respectively, and *RQ* parallel to *Pv*. In a Word, as all the Lines drawn here are Analogous to those in the Scheme for the correspondent Case in the Eclipse:

lipse; so from the Nature of these Curves will all the Reasonings here proceed after the same Manner as there. For here the Triangles PCF and Qvt are similar, because the Angles at F and t are Right Ones by the Hypothesis, and the Angle Qvt = PCF, because Qv is parallel to the Conjugate DC. From hence $Qt^q = \frac{Qv^q \times PF^q}{PC^q}$,

as before. Also (from *Conicks*) $Qv^q : Gv \times Pv :: CD^q : CP^q$, therefore $Pv = \frac{Qv^q \times CP^q}{Gv \times CD^q} = QR$.

From whence by Substitution of these Values of Qt^q and QR in the general Expression, viz.

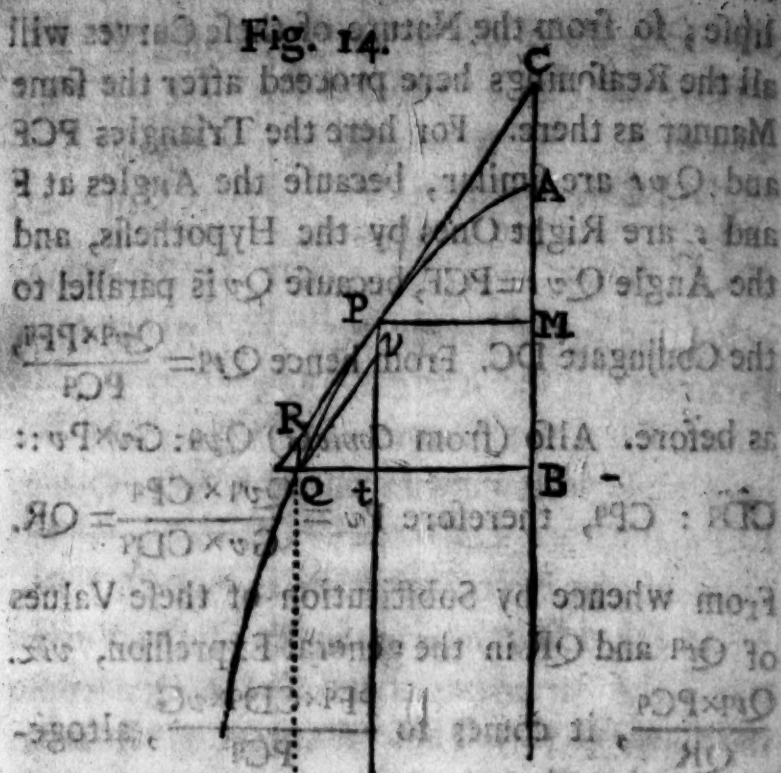
$\frac{Qt^q \times PC^q}{QR}$, it comes to $\frac{PF^q \times CD^q \times vG}{PC^q}$, altoge-

ther as in the Ellipse. Then substituting $BC^q \times CA^q$ for $PF^q \times CD^q$ (because $BC \times CA = CD \times PF$, from *Conicks*) and $2PC$ for vG , (when the Points P and Q come together;) Lastly, Dividing by the standing Quantity $2BC^q \times CA^q$, the Expression comes to $\frac{1}{PC}$. Therefore the

Force, with respect to the Point C, is reciprocally as $\frac{1}{PC}$, that is, directly as PC, the Di-

stance from the Center. Q. E. I.

Fig. 14.



P R O B. VIII.

If a Body moves in a Curve of a Parabola, 'tis requir'd to find the Law of the Centripetal Force tending to a Point infinitely distant.

LET AB (FIG. XIV.) be the Axe of the Parabola, RP a Tangent at P meeting the Axe in C, PM an Ordinate to the Axe, QS and PC Lines tending to the infinitely distant Center, and therefore parallel, and consequently Diameters of the Parabola; Qv an Or-

Ordinate to PS, Qr perpendicular to PS;
 Lastly, Let QS be produced to meet the Tan-
 gent in R. Now, because of the similar Tri-
 angles PCM, Qvr , 'tis $PC : PM :: Qv : Qr$,
 and so $Qr^q = \frac{PM^q \times Qv^q}{PC^q}$. Let the Parameter

belonging to the Vertex P be put $=R$, and the
 Parameter of the Axe $=r$, the Absciss AM $=x$
 Then, from the Nature of the Parabola, we
 have these following Equalities; viz. $Qv^q =$
 $R \times Pv$, $PM^q = rx$, $CM^q = 4x^2$, $PC^q = rx + 4xx$ (be-
 cause of the Rectangular Triangle PCM) $R =$
 $r + 4x$ (that is, the Parameter of the Vertex
 P is = the Parameter of the Axe + Quadruple
 the Absciss AM:) These Things are plain from
 the *Conick Elements*. Now substituting these
 Values in the Expression of Qr^q , we find

$$Qr^q = \frac{rx \times Qv^q}{rx + 4xx} = \frac{rx \times Pv \times R}{rx + 4xx} = \frac{rx \times Pv \times r + 4x}{rx + 4xx} =$$

$$\frac{Pv \times rrx + 4xx}{rx + 4xx} = Pv \times r \text{ (dividing by } rx + 4xx)$$

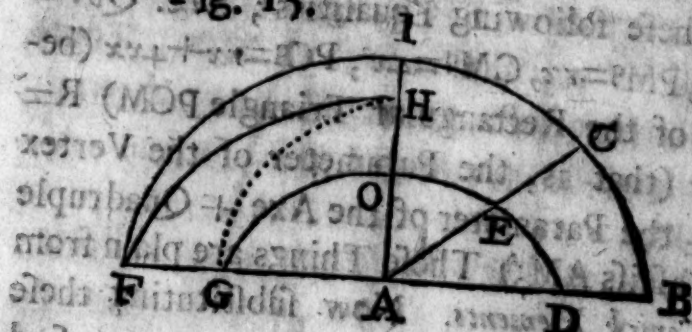
so that $Qr^q = Pv \times r$. Therefore the general Ex-
 pression of the Centripetal Force, viz. $\frac{PC^q \times Qr^q}{QR}$,

comes to $\frac{PC^q \times Pv \times r}{QR}$, which is (because $QR =$
 Pv from Parallels) $= PC^q \times r$. But r , the Para-
 meter of the Axe, is a standing Quantity, and
 because PC is infinite, therefore PC^q is ever a
 standing Quantity, and consequently the whole
 Expression is a standing Quantity, and there-

fore the Centripetal Force (which is reciprocally as $PC^2 \times r$) is in this Case equable. And this is *Galilei's Theorem*. Q. E. I.

Before we proceed to consider the Law of the Centripetal Force, when it tends to the *Foci* of the *Conick Sections*, 'tis proper to subjoin those other Propositions that relate to, and depend upon this present Law, in which the Force tends to the Center.

Fig. 15.



L E M M A.

Any Two Ellipses, as FIB, GOD, (FIG. XV.) having a common Center A, and the Transverse Axes FB, GD, lying in the same Right Line; the Centripetal Forces (tending to the common Center A) shall be in the Points F and G, as the Distances of those Points from the Center, viz. as $FA : GA$, that is, as the Semi-transverse Axes.

FOR upon the same Transverse FB of the outward Ellipse, one may still describe another Ellipse, as FH, whose Semi-conjugate AH

AH shall be equal to AG, the Semitransverse of the inward Ellipse. Now in the Ellipse FH, the Force in F is to that in H, as $FA : HA$ (by what has been shewn.) Also the Points H and G being equally distant from the Center A, the Forces in H and G are equal ; therefore the Force in F in the Ellipse FH, is to the Force in G in the Ellipse GO, as $FG : AG$ (which is $=AH$;) But the Force in F in the Ellipse FH, is equal to the Force in F in the Ellipse FI, since the Point F is common to them both. Therefore the Force in F in the Ellipse FI, is to the Force in G in the Ellipse GO, as $FG : AG$. Q.E. D.

THEOR. VII.

In all similar Ellipses, the Times of the Periodick Revolutions (perform'd about the same Center) are equal.

LET the Ellipses FIB, GOD, (FIG. XV. foregoing) be conceiv'd to be similar; then drawing any Diameter as AC, cutting the interior Ellipse in E, it might easily be demonstrated (from the supposed Similarity of the Figures) that $AD : AE :: AB : AC$, or that $AO : AE :: AI : AC$; and so that any other correspondent Diameters are still in the Proportion of the Axes. Now (the Centripetal Forces

in both Ellipses tending to the common Center A) the Force in B shall be to that in D, as $AB:AD$ (by the foregoing *Lemma*.) Therefore also the Force in C shall be to that in E, as $AC:AE$, that is, as $AB:AD$; and so drawing any other Diameter, the Centripetal Forces in the Two Ellipses shall still be in the same constant *Ratio* of the Semiaxes. But if Two Circles were describ'd upon the Center A with the *Radii* AB, AD, the Periodick Times in those Circles wou'd be equal; for the Times are equal (as was shewn in the *Corollaries* of *Theor. IV.*) when the Centripetal Forces are directly as the Rays, as here they are. And therefore since the Forces are in the same *Ratio* of these *Radii* AB and AD, every where in the Two Ellipses, the Periodick Times in them shall also be equal. Q. E. D.

S C H O L.

In the Demonstration foregoing, 'twas asserted, that the Centripetal Force in C is to that in E, as AC to AE; which is thus shewn. The Force in C is to the Force in B, as $AC:AB$, and the Force in E to that in D, as $AE:AD$ (from the general Law discover'd at *Prob. VI.*) But, from the Similarity of the *Figures*, 'tis $AC:AB::AE:AD$, therefore Force in C to Force in B, as Force in E to Force in D; and alternately, Force in C to Force in E, as Force in B

B to Force in D; but the Force in B and D are (by the foregoing Lemma) as AB: AD; that is, as AC, AE, therefore the Forces in C and E are also in the same Ratio of AC: AE.

LEMMA I

Putting the Parameter of the longer Axe of the Ellipse = L (FIG. , of Prob. VI.)

then shall $L = \frac{Qr^q \times PC^3}{QR \times PC^3}$ (taking this Quantity where the Points P and Q come together,

FOR it was found at that Prob. that $\frac{Qr^q \times PC^q}{QR}$ was = $\frac{2BC^q \times CA^q}{PC}$ But (from Conicks) the

Rectangle under the Parameter and the Semi-transverse, is equal to twice the Square of the Semi-conjugate, that is, $L \times CA = 2BC^q$, from whence $BC^q = \frac{L \times CA}{2}$, and so $\frac{L \times CA}{PC} = \frac{2BC^q \times CA^q}{PC}$

= $\frac{Qr^q \times PC^q}{QR}$. Therefore (by Reduction) L =

$\frac{Qr^q \times PC^q}{QR \times CA^q}$ Q. E. D.

LEMMA II

L E M M A II.

Supposing all as in the foregoing Lemma, (for the Centripetal Force tending to the Center of the Ellipse) I say, that the Area describ'd in a given Particle of Time, shall be express'd by this standing Quantity $L^{\frac{1}{2}} \times CA^{\frac{1}{2}}$.

FOR by the preceding Lemma, $L = \frac{Qr^q \times PC^3}{QR \times CA^3}$, therefore $Qr^q \times PC^3 = L \times QR \times CA^3$. But in a given Time, the Lineola QR is as the generating Centripetal Force (by the Second General Law of Motion) that is (in this Case) as PC, the Distance from the Center. Therefore since QR is as PC, then $Qr^q \times PC^3$ shall be as $L \times PC \times CA^3$, and (dividing by PC) $Qr^q \times PC^2$ shall be as $L \times CA^3$, and $Qr \times PC$ as $L^{\frac{1}{2}} \times CA^{\frac{3}{2}}$. But $Qr \times PC$ is as the Area describ'd in a given Time. Therefore this Area is as the Quantity $L^{\frac{1}{2}} \times CA^{\frac{3}{2}}$. Q. E. D.

C O R. T.

Putting L, l , the Parameters, and $2D, 2d$, the Conjugate or shorter Axes of Two Ellipses that have the same common Transverse $2CA$; then the Area's describ'd in these Ellipses in a given Time will be as the Conjugates. For the

Area's

Area's will be as $L^{\frac{1}{2}} \times CA^{\frac{3}{2}} : l^{\frac{1}{2}} \times CA^{\frac{3}{2}}$, that is (dividing by CA) as $L^{\frac{1}{2}} \times CA^{\frac{1}{2}} : l^{\frac{1}{2}} \times CA^{\frac{1}{2}}$, that is, as $2D : 2d$; for (from *Conicks*) $L \times CA = 2D^2$, $l \times CA = 2d^2$.

THEOR. VIII.

The Times of the Periodick Revolutions (performed about the same Center) are equal in all Ellipses that have the same Transverse Axis.

LET A be the *Area* of one Ellipse, T the Periodick Time, D the shorter Axis, N a Particle of the *Area* describ'd in a given Time; Let a be the *Area* of another Ellipse, t the Periodick Time, d the shorter Axis, n a Particle of the *Area* describ'd in the same given Time; and let B be the common Transverse Axis in both these Ellipses. Now, the *Area's* of Two Ellipses are in the *Ratio* compounded of the Periodick Times, and the Particles of the *Area's* describ'd together in a given Time. For the Periodick Time being the compleat Number of those Particles of the *Area*, that altogether compose the whole *Area* of the Figure; hence if those Particles are equal, the *Area's* would be as the Periodick Times, or if the Times were equal, the *Area's* would be as those Particles describ'd in a given Time; and therefore,

fore, universally speaking, the *Area's* are in the *Ratio* compounded of both. That is in Symbols, $A : a :: T \times N : t \times n$; therefore $T : t :: A \times n : a \times N$; that is, the Periodick Times are as the *Area's* directly, and the Particles describ'd together inverfly. But the Ellipses having their longer Axe common, the *Area's* of them are directly as the shorter Axes (which is evident from *Conick Sections*) that is, $A : a :: D : d$; so that now we have $T : t :: D \times n : d \times N$. But it was shewn at *Cor. of Lemma II.* foregoing, that in Ellipses having a common Transverse Axe, the Particles of the *Area's* described in a given Time were also as the shorter Axes directly; that is, $n : N :: d : D$. Therefore $T : t :: D \times d : d \times D$, which is a *Ratio* of Equality, so that $T = t$, or the Periodick Times in these Ellipses are equal. Q. E. D.

T H E O R. IX.

The Times of the Periodick Revolutions (perform'd about the same Center) are equal in all Ellipses whatsoever.

FOR by *Theor. VII.* the Times are equal in all similar Ellipses; and by *Theor. VII.* in all that have the same Transverse Axe. Therefore any Two Ellipses being given; how dissimilar soever they are, and how unequal soever their

their Transverse Axes be, yet it may presently be demonstrated, that the Periodick Times in them are equal. For (*FIG. XV.*) Suppose the Ellipses FH and GO were proposed; then A being their common Center, upon that same Center A, we may describe another Ellipse, as FI, that shall be similar to the Ellipse GO, and have the same Transverse with the Ellipse FH. Then the Periodick Times in GO and FI shall be equal, because they are similar, and the Times in FH and FI shall be equal, because they have the same Transverse Axe; therefore the Times in the Ellipses GO and FH shall also be equal: So that the Periodick Times in all Ellipses whatsoever are equal. Q. E. D.

T H E O R. X.

If a Body revolves in an Ellipse, and the Centripetal Force tends to the Center of the same; the Velocity in any Point as P (FIG. XV. Prob.) shall be express'd by this Quantity

$$\frac{L^{\frac{1}{2}} \times CA^{\frac{1}{2}}}{PF}, \text{ or (which is all one) it}$$

shall be in the Subduplicate Ratio of the

$$\text{Quantity } \frac{PF^3}{L \times CA}, \text{ Reciprocally (the Line PF}$$

being perpendicular from P, to the Diameter CF, which is a Conjugate to PC.)

FOR the Velocity in a given Time shall be express'd by the little Arch PQ describ'd in that Time; that is (since the little Arch and its Tangent are at last coincident) by the Tangent PC, or because of the Parallelogram QRPv) by the Lineola Qv. But the Triangles Qvt, PCF being similar, the Lineola Qv = $\frac{PC \times Qt}{PF}$; also PC x Qt is as the Area describ'd

in a given Time. But (by Lemma II.) the Area describ'd in a given Time is express'd by

$$L^{\frac{1}{2}} \times CA^{\frac{1}{2}}, \text{ therefore Qv is as } \frac{L^{\frac{1}{2}} \times CA^{\frac{1}{2}}}{PF}; \text{ that}$$

is,

is, the Velocity is in the subduplicate Ratio
of $\frac{PF^2}{L \times CA^2}$ Reciprocally. Q. E. D.

That the *Corollaries* may be more easily apprehended, I shall premise some preparatory Interpretations of this general Expression of the Velocity. In order to which, let $2D$, $2d$ be the shorter Axes, L , l the Parameters (of the longer Axes of any Two Ellipses) P , p the Perpendiculars from the Centers to the Tangents; or, which is all one, the Lines PF , mention'd just now: The general Expression now,

viz. $\frac{L^{\frac{1}{2}} \times BA^{\frac{1}{2}}}{PF}$, in these more compendious

Terms comes to $\frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{P}$, or $\frac{l^{\frac{1}{2}} \times b^{\frac{1}{2}}}{p}$, for one and the other Ellipse. And to determine this to particular Points of the Curve (*Ex.gr.*) in the Extremities of the longer and shorter Axes, we need only consider, that in those Extremities the Perpendiculars P , p are coincident with the half longer and shorter Axes themselves. Therefore,

1. In the Extremity of the shorter Axe, the Expression comes to $\sqrt{2 \times B}$. For in this

Case P is $= D$ = to half the shorter Axe $= \sqrt{\frac{L \times B}{2}}$

$= \frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{\sqrt{2}}$, as is plain from the Nature of the

Ellipse. Therefore $\frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{P}$ ($= \frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{D}$) is $=$
 $\frac{\sqrt{2} \times L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{L^{\frac{1}{2}} \times B^{\frac{1}{2}}} = \sqrt{2} \times B.$

2. In the Extremity of the longer Axe, the Expression comes to $L^{\frac{1}{2}} \times B^{\frac{1}{2}}$. For here in this Case, P is $= B$ = to half the longer Axe.

3. For a Circle whose *Radius* is $= \frac{1}{2}$ the shorter Axe (putting R the *Radius*) the Expression of the Velocity comes to $\sqrt{2} \times R$ in the Terms of the Circle; and to $L^{\frac{1}{2}} \times B^{\frac{1}{2}}$ in the Terms of the Ellipse. The former part is clear: For in the Circle L is $= 2R$, $B = R$, and $P = R$, therefore $\frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{P}$ is $= \sqrt{2} \times R$. The latter part appears, in that R is $= D$ by the *Hypothesis*, that is, $R = \frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{\sqrt{2}}$, and so $R \times \sqrt{2} = L^{\frac{1}{2}} \times B^{\frac{1}{2}}$ in the Terms of the Ellipse.

4. For a Circle whose *Radius* is $\frac{1}{2}$ the longer Axe, the Expression of the Velocity comes (as before) to $\sqrt{2} \times R$ in the Terms of the Circle; and to $\sqrt{2} \times B$ in the Terms of the Ellipse. For in this Case R is $= B$, and so $\sqrt{2} \times R$ is $= \sqrt{2} \times B$. Now from hence the following *Corollaries* will appear to be very clear.

COR.

C O R. I.

In similar Ellipses, and in Points similarly posited to the principal Vertexes, the Velocities are every where in some standing Ratio of the longer or shorter Axes, or the Parameters.

C O R. II.

In Ellipses that have the same Transverse Axe, the Velocities are as the square Roots of the Parameters directly, and the Perpendiculars from the common Center to the Tangents

inversly. For the Expressions are $\frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{P}$, and $\frac{l^{\frac{1}{2}} \times b^{\frac{1}{2}}}{p}$; therefore if $B = b$, the Velocities in any

Two such Ellipses are as $\frac{L^{\frac{1}{2}}}{P} : \frac{l^{\frac{1}{2}}}{p}$.

C O R. III.

The Transverse Axe being common, the Velocities in the Two Ellipses in the Extremity of the Transverse Axe, are as the shorter Axes directly. For in this Case B is $= b$, and the Perpendiculars P, p . do both coincide with B or b ; therefore the Velocities are as $L^{\frac{1}{2}} : l^{\frac{1}{2}}$. But because $B = b$, therefore $L^{\frac{1}{2}} : l^{\frac{1}{2}} :: L^{\frac{1}{2}} \times B^{\frac{1}{2}} : l^{\frac{1}{2}} \times b^{\frac{1}{2}}$, which

which Expressions are as the shorter Axes ;
therefore, &c.

C O R. IV.

Supposing all as before, the Velocities in the
Extremities of the shorter Axes are equal : For
P and p are here coincident with D and d,
also, by the *Hypothesis*, B is = b, therefore the
Expressions of the Velocities in the Two El-
lipses amount to $\frac{L^{\frac{1}{2}}}{D}$, and $\frac{l^{\frac{1}{2}}}{d}$; that is (putting
in the Values of D and d) to $\frac{\sqrt{2 \times L^{\frac{1}{2}} \times B^{\frac{1}{2}}}}{L^{\frac{1}{2}}}$, and
 $\frac{\sqrt{2 \times l^{\frac{1}{2}} \times b^{\frac{1}{2}}}}{l^{\frac{1}{2}}}$, which are equal, because B=b.

C O R. V.

In one and the same Ellipse, the Veloci-
ties are Reciprocally as the Perpendiculars
from the Center, to the Tangents at any
Points where the Body is supposed to be.
For the Expression $\frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{P}$ (the Numerator be-
ing a standing Quantity, and consequently
made equal to Unity) comes to $\frac{1}{P}$. So that the
Velocity in the Extremity of the shorter Axe,
is to the Velocity in the Extremity of the longer
Axe, as the longer Axe is to the shorter.

C O R.

C O R. VI.

The Velocity in the Extremity of the shorter Axe, is to the Velocity in a Circle at the same Distance, as the longer Axe is to the shorter. For (by the first *Preliminary* foregoing) the Expression of the Velocity for the Extremity of the shorter Axe is $\sqrt{2 \times B}$; and (by *Prelim. III.*) that for a Circle whose *Radius* is $= \frac{1}{2}$ the shorter Axe, is $\sqrt{L \times B}$. But $\sqrt{2 \times B} : \sqrt{L \times B} :: B : \sqrt{\frac{L \times B}{2}}$; which Proportion is manifest, and the Two latter Terms are the half longer and shorter Axes. Therefore, &c.

C O R. VII.

The Velocity in the Extremities of the longer Axe, is to the Velocity in a Circle at the same Distance, as the shorter Axe is to the longer. For (by the 2d *Prelim*) the Expression for the Extremity of the longer Axe is $\sqrt{L \times B}$, and (by *Prelim. IV.*) that for a Circle whose *Radius* is $=$ half the longer Axe, comes to $\sqrt{2 \times B}$, the former of which Expressions is evidently to the latter, as the shorter Axe to the longer.

C O R. VIII.

Comparing the Accounts of the Two last *Corollaries*, it will be found, that the Expressions of the Velocities in the Ellipse at the Extremity of the shorter Axe, and in a Circle whose *Radius* is = half the longer Axe, are the same; and likewise of the Velocity in the Ellipse at the Extremity of the longer Axe, and that in a Circle whose *Radius* is = half the shorter Axe, are the same, *viz.* for the Two former $\sqrt{2 \times B}$, and for the Two latter $\sqrt{L \times B}$; so that the Velocities in these Circles, and these Points of the Ellipse are equal.

C O R. IX.

Two Ellipses having a common Center, and their longer Axes lying in the same Right Line; if they are so proportion'd to each other, that the Parameters of the longer Axes be reciprocally as those Axes, then the Velocities in the Extremity of the shorter Axe shall be in each Ellipse, as the longer Axes directly. It must be observ'd here, that these Ellipses thus proportion'd to one another shall have the shorter Axe common to them both. For, by the Supposition, 'tis $L : l :: b : B$, from

whence $L \times B = l \times b$, and so $\sqrt{\frac{L \times B}{2}} = \sqrt{\frac{l \times b}{2}}$, or
 $D = d$.

$D=d$. Now the Expressions of the Velocities in the Extremity of the shorter Axe, for each Ellipse are $\frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{D}$, and $\frac{l^{\frac{1}{2}} \times b^{\frac{1}{2}}}{d}$, that is (putting in the Value of D and d) $\sqrt{2 \times B}$, and $\sqrt{2 \times b}$, which are as B and b . Therefore, &c.

C O R. X.

Supposing all as in the last *Corollary*, the Velocities in the Extremities of the longer Axes are equal to one another : For in this Case the Expressions amount to $\frac{L^{\frac{1}{2}} \times B^{\frac{1}{2}}}{B}$, and $\frac{l^{\frac{1}{2}} \times b^{\frac{1}{2}}}{b}$, that is, to $L^{\frac{1}{2}} \times B^{\frac{1}{2}}$, and $l^{\frac{1}{2}} \times b^{\frac{1}{2}}$, or $\sqrt{L \times B}$ and $\sqrt{l \times b}$; but these Expressions are equal, because $L \times B$ is $= l \times b$, as was shewn in the last *Corollary*. Therefore, &c.

C O R. XI.

Comparing *Corolls.* the 3d and 4th with *Corolls.* the 9th and 10th, we may observe a Parallel not unworthy Consideration. For (by *Cor.* III.) the longer Axe being common, the Velocities in the Extremities of the longer Axe are as the shorter Axes directly; and (by *Cor.* IX.) the shorter Axe being common, the Velocities in the Extremity of the shorter Axe are as the longer Axes directly. Again, (by *Cor.* IV.) sup-

posing what was supposed in *Cor. III.* the Velocities in the Extremities of the shorter Axes are equal; and (by *Cor. X.*) supposing what was supposed in *Cor. IX.* the Velocities in the Extremities of the longer Axes are also equal to one another.

C O R. XII.

If Two Ellipses be so proportion'd to one another, that the shorter Axe of the one be equal to the longer Axe of the other, then the Velocity in the Extremity of the longer Axe of the one Ellipse shall be equal to the Velocity in the Extremity of the shorter Axe of the other. For the Expression of the Velocity in the Extremity of the longer Axe of one

Ellipse is $\frac{L^2 \times B^3}{B} = L^2 \times B$; and that for the Ve-

locity in the Extremity of the shorter Axe

of the other, is $\frac{l^2 \times b^3}{d} = \frac{\sqrt{2 \times l^2 \times b^3}}{l^2 \times b^2} = \sqrt{2 \times b}$.

Now, if $L \times B = 2bb$, or $\sqrt{L \times B} = \sqrt{2 \times b}$, or $\sqrt{\frac{L \times B}{2}}$

(the Semi-conjugate of the one) be = b (the Semi-transverse of the other) then 'tis plain, that these Expressions of the Velocities are equal. Or we may say it will be so, if it be $L : 2b :: b : B$; if the Parameter of one Ellipse be to the Transverse of the other, as the Semi-

Semi-transverse of that other to the Semi-transverse of the first.

C O R. XIII.

If the Center of the Ellipse C be imagin'd to remove to an infinite Distance, and so the Ellipse to change into a Parabola, a Body shall move in the same with an equable Velocity. For in this Case all the Quantities that enter the general Expression, viz. $\frac{L^2 \times B^2}{P}$, are standing ones, the Quantities B and P being both infinite, and L being the Parameter of the Axe. And here also (*mutatis mutandis*) one may apply what has been said in the *Corollaries* foregoing of the true Ellipse to the Parabola, or Ellipse whose Center is at an infinite Distance.

C O R. XIV.

And if this Parabola (by changing the Inclination of the Plane cutting the Cone) degenerates into an Hyperbola, the Body shall move in the same with a Velocity, which shall be reciprocally as the Perpendicular from the Center to the Tangent, as was shewn for the Ellipse at *Cor. V.*

I proceed now to the Consideration of the Law of the Centripetal Forces and Velocities
of

of Bodies, when those Forces are directed to the *Foci* of the *Conick Sections*.

P R O B. IX.

A Body revolving in an Ellipse, (FIG. XI.) 'tis requir'd to find the Law of the Centripetal Force tending to the Focus.

HERE the *Lineola* QR is supposed to be Parallel to Sp, so that QRxP is a Parallelogram; also the *Lineola* QT is a Perpendicular to SP, PF perpendicular to the Conjugate DK; the Points S and H are the *Foci*, and all the rest as at *Prob. VI.* before. It must be premised before we proceed to the Investigation with the admirable Author, that EP (cut off by CD, the Conjugate to PG) is equal to AC, half the longer Axe. For drawing IH from the other *Focus* H parallel to DC, we have SC = CH from the Nature of the *Foci*, and ES = EI, because of the Parallels EC and IH, Again, since $SP = IP + 2EI$, and consequently $IP + EI$ (that is EP) = $SP - EI$; 'tis plain, that
$$EP = \frac{SP - EI + IP + EI}{2} = \frac{SP + IP}{2}.$$
 But now, from *Conick Sections*, the Angles HPZ, IPR, are equal, and therefore their Alternates PHI, PIH are equal also (for IH parallel to DK, by the *Hypothesis*, is parallel also to the Tangent PZ at

at the Point P of the Diameter PC, Conjugate to DK.) Therefore $PI=PH$, and consequent-

ly $EP (= \frac{SP+IP}{2})$ is $= \frac{SP+PH}{2}$, which (from

Conick Sections) is equal to CA, which is half the longer Axe of the Ellipse. This premised, we proceed to the Investigation of the Law of the Centripetal Force.

The Triangles QTx, PFE are similar (for the Angles T and F are Right ones, and Angle $x =$ Angle E, because of the Parallels Qx and DF) so that $QT^q = \frac{Qx^q \times PF^q}{CA^q}$; and from the

similar Triangles Pxv, PEC, 'tis $QR (=Px)$
 $\frac{Pv \times PE}{PC} = \frac{Pv \times AC}{PC}$, because $EP=CA$. Therefore

by Substitution $\frac{SP^q \times QT^q}{QR} = \frac{SP^q \times Qx^q \times PF^q \times PC}{Pv \times CA^3}$

But, from *Conick Sections*, $PC^q : CD^q :: Gv^q \times Pv^q :$

Qv^q , consequently $Qv^q = \frac{Gv \times Pv \times CD^q}{PC^q}$; also

when the Points P and Q come together, $Qx=Qv$, therefore instead of Qx^q , putting in this Value of Qv^q in the foregoing Expression, it

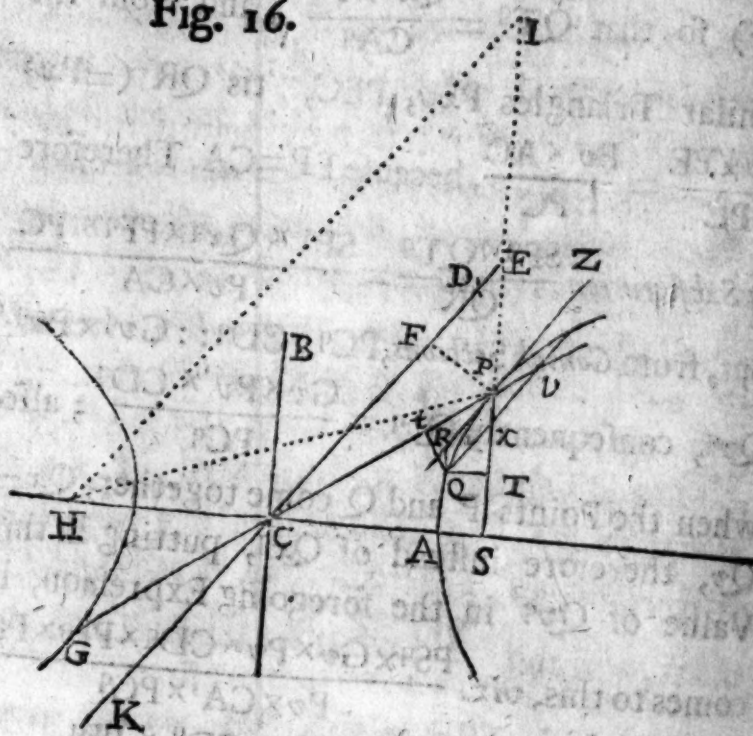
comes to this, viz. $\frac{PS^q \times Gv \times Pv \times CD^q \times PF^q \times PC}{Pv \times CA^3 \times PC^q}$

$= \frac{SP^q \times Gv \times CD^q \times PF^q}{CA^3 \times PC} = \frac{SP^q \times 2CD^q \times PF^q}{CA^3}$, be-

cause when the Points P and Q come together,
 $Gv=2PC$.

$Gv = 2 PC$. But from *Conicks* $2 CD_q \times PF_q = 2 BC_q \times CA_q$, therefore the Expression comes to this, viz. $\frac{SP_q \times 2 BC_q \times CA_q}{CA^3} = \frac{SP_q \times 2 BC_q}{CA} = SP_q \times L$ (because $L = \frac{2 BC_q}{CA}$) so that the Centripetal Force, which comes to this Expression $SP_q \times L$, dividing by L , a standing Quantity, being the Parameter of the Axe) is reciprocally as SP_q that is, as the Square of the Distance from the Focus. Q. E. I.

Fig. 16.



PROB.

P R O B. IX.

A Body revolving in an Hyperbola, 'tis requir'd to find the Law of the Centripetal Force tending to the Focus.

HERE AC and CB (FIG. XVI.) be the Semiaxes, PG, KD Two Diameters conjugate to each other, Qr and PF Perpendiculars to those Diameters respectively, Qv an Ordinate to PG. The Points S and H are the *Foci*, and from S is drawn SP, cutting the Diameter DK in E, and the Ordinate Qv in x, and the Figure QRPx is supposed to be a Parallelogram. All the rest as before in the Ellipse, and other Figures drawn for the same Purpose. Now, it must be premised here also, that the Line EP (cut off by KD, the Conjugate to PG) is = AC, the Semi-transverse Axe: For drawing HI from the other *Focus* H, parallel to DC, we have SC=CH from the Nature of the *Foci*; also ES is EI, because of the Parallels EC and IH. Farther, since $2EI = SP + IP$, and $EI - SP (=EP) = IP - EI$; 'tis evident, that EP is = $\frac{IP - EI + EI - SP}{2} = \frac{IP - SP}{2}$.

But now, from *Conick Sections*, the Angles IPZ and HPR are equal, and consequently their Alternates HIP, IHP are equal also (for IH parallel

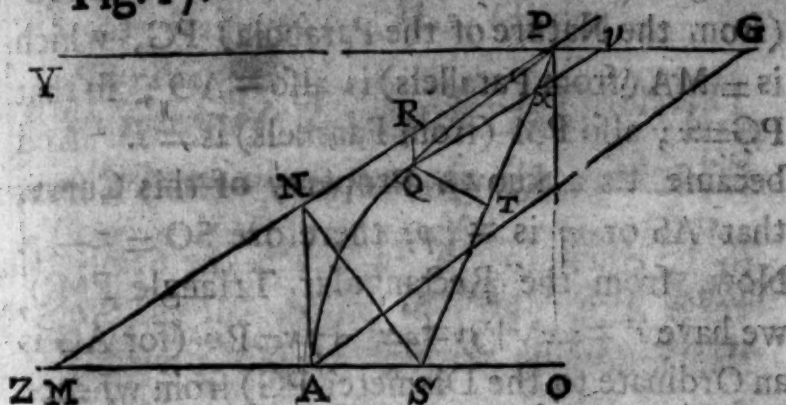
parallel to the Diameter DK, by the *Hypothesis*, is parallel also to the Tangent PZ at the P of the Diameter PC, Conjugate to DK.)

Therefore $PH = PI$, and so $EP = \frac{PI - SP}{2}$, is = $\frac{PH - SP}{2}$, which (from *Conick Sections*) is known

to be equal to CA, which is the Semi-transverse Axe. This premised, all the Steps of the following Process are so perfectly the same with these in the Ellipse, both as to the Substitutions, and the Order and Dependence of one Step upon another, that I need not make the tedious Repetition, but leave the industrious Reader to draw it out himself, which he cannot miss of doing, understanding but the easie Process in the foregoing *Problem*. And the Conclusion here will be, as there, that the Centripetal Force is Reciprocally, as SP^4 , that is the Square of the Distance from the *Focus*. Q. E. I.

LEMMA.

Fig. 17.



LEMMA I. (Prob.)

In the Parabola, any Vertex (or Point thro' which the Diameter passes) being given, to find the Proportion the Parameter belonging to that Vertex bears to the Distance of the same Vertex from the Focus.

LET (FIG. XVII.) AO be the Axe of the Parabola, PO an Ordinate, S the Focus, from which is drawn SP to the Point P, and thro' the same the Diameter YPG. Let PM touch the Curve at P, and AG be drawn parallel to PM.

Let $AO = x$. $AG = s$. $SP = z$. $AS = m$. Parameter of the Axe $= p$. Parameter of the Diameter $PG = P$. The Figure AMPG is a Parallelogram, for PG is a Diameter, and therefore parallel

to

to AO, and AG is, by the *Hypothesis*, parallel to PM. Therefore since MA is = AO (from the Nature of the Parabola) PG, which is = MA (from Parallels) is also = AO; so that $PG=x$; also PM (from Parallels) is = s . And because 'tis a known Property of this Curve, that AS or m is = $\frac{1}{4}p$, therefore $SO = z - \frac{1}{4}p$. Now, from the Rectangular Triangle PMO, we have $s^2 = 4x^2 + yy = 4x^2 + px = Px$ (for AG is an Ordinate to the Diameter PG) from whence $P=4x+p$. Again, from the Rectangular Triangle SPO, we have $z^2 = yy + xx - \frac{1}{2}px + \frac{1}{16}$

$pp = \frac{px}{2} + xx + \frac{1}{16}pp$ (putting in for yy its Value px) consequently $z = x + \frac{1}{4}p$. But P was found $= 4x + p$, therefore $P = 4z$. That is, the Parameter belonging to Diameter PG, is Quadruple of PS, the Distance from the Vertex to the *Focus*. Q. E. I.

The Result will be found the same, if the Ordinate PO be taken on the other Side the *Focus* S; so that SO instead of $x - \frac{1}{4}p$, becomes $\frac{1}{4}p - x$.

C O R.

It is plain, that PS is = MS, for each is $= x + \frac{1}{4}p$; and therefore also the Parameter P is Quadruple of MS.

LEMMA

L E M M A II. (Prob.)

In the foregoing Figure the Line SN being a Perpendicular from the Focus S to the Tangent PM , 'tis requir'd to find the Proportion of SP (the Distance from the Focus to the Point of Contact P) to the Perpendicular SN .

ALL the other Symbols standing as before, let $SN=f$. By the Corollary of the foregoing Lemma, it appears, that $PS=MS$, from whence it follows that $PN=MN$, or $PN=\frac{1}{2}PM$, or $PN^2=\frac{1}{4}PM^2=\frac{1}{4}px+xx$. Therefore from the Rectangular Triangle SNP , we have $f^2=$

$$x^2+\frac{px}{2}+\frac{1}{16}pp=\frac{1}{4}px+xx=\frac{1}{4}px+\frac{1}{16}pp. \text{ So}$$

that $x+\frac{1}{4}p:f::f:\frac{1}{4}p$, that is, f (the Perpendicular from the Focus to the Tangent) is a mean Proportional between the Quantities $x+\frac{1}{4}p$ and $\frac{1}{4}p$, that is between the Distances of the Point of Contact, and of the principal Vertex from the Focus, or in other Terms, that AS
 $SN::SN:SP.$ Q. E. D.

Cor. of Lemma I. foregoing, $PS=MS$; therefore $PM=PS$. This premised, let the Parameter be called p . Then

C O R.

Hence because of $\frac{1}{4}p$, or AS a standing Quantity, f^2 , or SN^2 , is always as $x + \frac{1}{4}p$, or SP: That is, the Distance from the Focus PS is ever as the Square of the Perpendicular from the Focus to the Tangent.

P R O B. XI.

A Body revolving in a Parabola, 'tis requir'd to find the Law of the Centripetal Force tending to the Focus.

THE rest of the Construction remaining as in the foregoing Lemma, let QR be parallel to SP, QT perpendicular to the same, Qv parallel to the Tangent PM, cutting SP in x , and YPG in v . Before I proceed to the Investigation, 'tis necessary to premise, that P x is Px, which is thus prov'd. The Angle PMS (= MPY, as being Alternates) is = Angle P ox , because of the Parallels MS and PG; and the Angle vPx is = PSM, because of Parallels. Therefore the Triangles MPS and P xv are similar, and so $MS : SP :: Px : Pv$, but by Cor. of Lemma I. foregoing, PS is = MS; therefore $Px = Pv$. This premised, let the Parameter of the Diameter PG be call'd L. Then

(from

(from the Nature of the Parabola) $L \times Pv = Qv^2$
 $= 4 SP \times Pv$ (because $L = 4 SP$, by Lemma I. foregoing) $= 4 SP \times Px$ (because $Pv = Px$, as was just
 now shewn) $= 4 SP \times QR$ (because the Figure $QRPx$ is a Parallelogram, by the Construction.)
 But when the Points P and Q come together,
 the *Lineola* Qv will be coincident with or equal
 to Qx ; therefore in that Case Qx^2 shall =
 $4 SP \times QR$. Lastly, Because the Triangles QxT ,
 SPN are similar (for the Angles QTx , and
 SNP are Right ones, and $QxT = NPS$ from Pa-
 rallels (therefore $Qx^2 : QT^2 :: PS^2 : SN^2$; but
 (by Lemma II. foregoing) 'tis $PS : SN :: SN : AS$,
 and consequently $PS^2 : SN^2 :: PS : AS$;
 therefore (by Equality of Proportion) $Qx^2 : QT^2 :: PS : AS$, and (multiplying PS and AS
 by $4 QR$) 'tis $Qx^2 : QT^2 :: 4 SP \times QR : 4 AS \times QR$.
 But $Qx^2 = 4 PS \times QR$; therefore $QT^2 = 4 AS \times QR$.
 Therefore substituting this Value of QT^2 in
 the general Expression $\frac{SP \times QT^2}{QR}$, it amounts to
 $SP \times 4 AS$, and $4 AS$ being = the Parameter of
 the Axe, and so a standing Quantity, it comes
 to SP^2 . So that the Centripetal Force is Reci-
 procally as SP^2 ; that is, as the Square of the
 Distance from the Focus. Q. E. I.

COROLLARIES to the preceding PROBLEMS concerning the Law of the Centripetal Force tending to the Foci of the Conick Sections.

C O R. I.

It follows, that if a Body moves from any Place, as P, and is drawn by a Centripetal Force Reciprocally, proportional to the Square of the Distance from the Center to which those Forces are directed; that then it moves in a Conick Section, whose Focus is coincident with that Center.

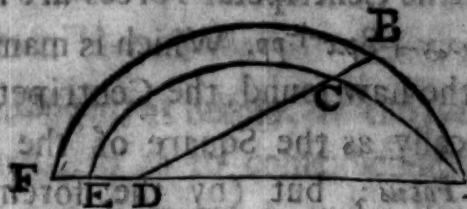
C O R. II.

And if the Velocity with which the Body goes out of the Place P be such, that while the *Lineola* PR is described by it, the Centripetal Force carries it thro' the *Lineola* QR in the same Time, then it moves in a Conick Section, whose Parameter is $= \frac{QT^2}{QR}$ when the Points P and Q come together.

And, with respect to the particular Sections, from the Law found we may infer,

C O R.

Fig. 18.



C O R. III.

That if Two Ellipses, as FBA, ECA (FIG. 18.) have their Vertex A, and one Focus (as the Point D) common to them both, that then the Centripetal Force in C shall be to that in B (the Focus D being the Center of the Forces) as $BD^2 : CD^2$. For putting the Force in A = F, that in C = f, that in B = ϕ , 'tis plain, that

$\frac{f}{\phi} = \frac{f}{F} \times \frac{F}{\phi}$, but $\frac{f}{F} = \frac{AD^2}{CD^2}$, and $\frac{F}{\phi} = \frac{BD^2}{AD^2}$, therefore $\frac{f}{\phi} = \frac{BD^2}{CD^2}$ and so $f : \phi :: BD^2 : CD^2$.

That the Two Ellipses may be such as is required, the Lines FD, ED must be in the duplicate Ratio of the shorter Axes, the Mention of which is sufficient.

C O R. IV.

In the Parabola, the Centripetal Forces are Reciprocally, as the Squares of the Parameter of the Axe increased by Quadruple the Absciss (taken in the Axe) for any Point of

the Curve. That is (in the Terms of *Lemma I.* foregoing) the Centripetal Forces are Reciprocally as $16xx + 8px + pp$. Which is manifest, because by the Law found, the Centripetal Force is Reciprocally as the Square of the Distance from the *Focus*; but (by the foremention'd *Lemma*) the Expression of any Distance from the *Focus* is $x + \frac{1}{4}p$. Therefore, &c.

'Tis also equally true, that the Centripetal Forces are Reciprocally as the Squares of the Parameters belonging to the several Vertices, or Points where the Body is. For those Parameters are Quadruple the respective Distances from the *Focus*.

The Law of the Centripetal Force tending to the *Focus* of the Ellipse being discover'd, the Application may easily be made to the rest of the *Conick Sections* without a particular Investigation, as the celebrated Author himself observes. (See *FIG. XI.*)

If the *Focus S* of the Ellipse to which the Centripetal Force tends, continues together with the Vertex *B*, while the other *Focus H* removes to an infinite Distance, the Ellipse now is converted into a Parabola; but the Body will still move into a Perimeter of this Curve, and the Law of the Centripetal Force tending to the

the *Focus* S continues what it was before, viz. Reciprocally as the Square of the Distance from S.

Supposing all as before, if the *Focus* H moves backwards, so as to come behind the Vertex B, both the Vertices A and B, lying now between the *Foci* S and H, the Ellipse becomes an Hyperbola, and the Body moving in the Perimeter of this Curve, the Law of the Force tending to the *Focus* S continues the same as at first.

If the *Focus* H and Vertex A be supposed fix'd, and S, to which the Forces tend, removes to an infinite Distance, the Ellipse is changed into a Parabola, and the Forces tending to a Point infinitely distant are all equal.

The *Focus* H and Vertex A continuing, if the *Focus* S, to which the Forces tend, moves backwards, as in Case II. then we have an Hyperbola again, in the Periphery of which the Body shall still move, but with a Centrifugal Force (instead of a Centripetal) which shall be according to the establish'd Law, Reciprocally as the Square of the Distance from S.

If the Two *Foci* S, H come together, the Ellipse becomes a Circle, and the Force tending to the Center is equable.

For P 4 If

If the *Foci* S, H coincide with the Vertices B, A, the Ellipse degenerates into a Right Line; in which Case there's no Centripetal Force tending to any Point without the Line it self.

T H E O R. XI.
Bodies revolving about a common Center, and the Centripetal Force being in a Reciprocal duplicate Ratio of the Distances from the Center; then the Latera Recta of the Orbs describ'd shall be in the duplicate Ratio of the Area's which the Bodies describe in the same Time, by Rays drawn to that Center.

F O R. (by *Cor. II. of Prob. XI.*) $L = \frac{QT^q}{QR}$; but QR in a given Time is as the Centripetal Force (by the *Second Law of Motion*) that is, as SP^q Reciprocally (by *Prob. IX.*) Therefore $\frac{QT^q}{QR}$ is as $QT^q \times SP^q$, that is, L is as $QT^q SP^q$; but the Area is as $QT \times SP$. Ergo, &c. **Q. E. D.**

C O R.

The whole Area's of Ellipses are in the Ratio compounded of the $\frac{1}{2}$ Ratio of the *Latera Recta*, and the Ratio of the Periodick Times
 For

For those *Area's* are in the *Ratio* compounded of the *Ratio* of the Portions describ'd in the same Time, and that of the Periodick Times: But by this *Prop.* those Portions are in the subduplicate *Ratio* of the *Lateral Recta*. Therefore, &c.

And since (by *Prop.* 191. of *Gregory St. Vincent*) the *Area's* of Ellipses are as the Rectangles under their Axes, it's certain, that those Rectangles are also in the same *Ratio*.

THEOR. XIII.

Supposing all as before, the Periodick Times in Ellipses are in the $\frac{1}{2}$ (the Sesquiplicate) Ratio of the Transverse Axes.

FOR reassuming the Symbols as in *Theor.* 8. we find that $T : t :: A \times n : a \times N$; but $A : a :: D \times B : d \times b$ (by the foremention'd *Theor.* of *St. Vincent*) therefore $T : t :: D \times B \times n : d \times b \times N$. But $D : d :: L^{\frac{1}{2}} \times B^{\frac{1}{2}} : l^{\frac{1}{2}} \times b^{\frac{1}{2}}$, from *Conicks*; and $n : N :: L^{\frac{1}{2}} : l^{\frac{1}{2}}$, by the foregoing *Theorem*, therefore $T : t :: L^{\frac{1}{2}} \times B^{\frac{1}{2}} \times B \times l^{\frac{1}{2}} : l^{\frac{1}{2}} \times b^{\frac{1}{2}} \times b \times L^{\frac{1}{2}}$, that is (dividing by $L^{\frac{1}{2}} \times l^{\frac{1}{2}}$) $T : t :: B^{\frac{1}{2}} : b^{\frac{1}{2}}$. The Periodick Times in Ellipses are in the Sesquiplicate Ratio of the Transverse Axes. Q. E. D.

COR.

C O R.

The Periodick Times in Ellipses are the same with those in Circles, whose Diameters are equal to the longer Axes. For by the Converse of Case IV. from the Corollaries of Theor. III. this Proportion of the Squares of the Times and the Cubes of the Rays belongs to Circles, when the Law of the Centripetal Force is the same that 'tis here, viz. Reciprocally as the Square of the Distance from the Center to which the Forces are directed.

T H E O R. XIII.

The Centripetal Force tending to the Focus, as before; PR being a Tangent at P, the Place of the Body, and ST a Perpendicular from the Focus to it, the Velocity of the Body in P shall be in the 1^{st} Ratio of $\frac{ST^2}{L}$

Reciprocally. (See FIG. XI.)

FOR the Velocity is as the little Ark PQ describ'd in a given Time; that is, as the Tangent PR, or because of Parallels, as Qx. But, from similar Triangles, QTx, SYP, Qx

or

or $PR = \frac{SP \times QT}{SY}$; therefore PQ is as SY Recipro-

procally, and $SP \times QT$ directly; that is, as SY Recipro-
 cally, and the *Area* describ'd in a given
 Time directly; but (by *Theorem XI*) $SP \times QT$

is as L , therefore PQ is as $\frac{L}{SY}$, that is, in the

$\frac{1}{2}$ Ratio of $\frac{SY}{L}$ Reciprocallly. Q. E. D.

In order to the Investigation of the many
 noble *Corollaries* that follow from this *Theorem*,
 let these perpetual Symbols stand.

SP (the Distance from the *Focus* to any Point
 in the Curve) = z . SY (the Perpendicular from
 the *Focus* to the Tangent at the same Point) = y .

The Velocity in a *Conick Section* at the Distance
 z , put = V . The Velocity in a Circle at the
 same Distance z , put = W . The Velocity in
 a Circle (whose *Radius* is equal to any other
 Distance, whether greater or less than z) put

= u . The longer *Axe* of any *Ellipse* put
 = $2B$, the shorter = $2D$, the Parameter = L ;
 and this Symbol L I use also for the Parameter
 of any *Conick Section* in general, as the Occa-
 sion requires.

Now

Now then, the Expression of the Velocity in a

Conick Section (at any Distance as z) is $\frac{L^{\frac{1}{2}}}{y}$; that

is, V is ever as $\frac{L^{\frac{1}{2}}}{y}$. Likewise the Expression

of the Velocity in a Circle at the Distance

z wou'd be $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$. For the *Radius* being

$= z$, then L in the Circle is $= 2z$, or the

Diameter; and y the Perpendicular from the

Focus to the Tangent, being in the Circle $=$

the *Radius*, that is $= z$: Hence in the Terms

of the Circle $\frac{L^{\frac{1}{2}}}{y}$ is $\frac{\sqrt{2z}}{z}$, or $\frac{\sqrt{2 \times z^{\frac{1}{2}}}}{z^{\frac{1}{2}}} = \frac{\sqrt{2}}{z^{\frac{1}{2}}}$.

So that then W is as the Expression $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$, or $\sqrt{\frac{2}{z}}$.

Now for the *Corollaries* themselves.

C O R. I.

Because V is as $\frac{L^{\frac{1}{2}}}{y}$, therefore $L^{\frac{1}{2}}$ is as Vy ,

and L is as V^2y^2 . That is, the Parameters of

the Orbs are in the *Ratio* compounded of the

duplicate *Ratio* of the Velocities, and the du-

plicate *Ratio* of the Perpendiculars from the

Focus to the Tangents.

C O R. II.

When the Quantity y (in the Expression of the Velocity) becomes coincident with the greatest or least Distance from the *Focus*; 'tis manifest then, that in those Points of greatest or least Distance, the Velocities are as those Distances inverfly, and the square Roots of the Parameters directly.

C O R. III.

Because V is as $\frac{L^{\frac{1}{2}}}{y}$, and W as $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$, therefore if $y = z$ (which cannot be but when z denotes the greatest or least Distance) then the Expressions are $\frac{L^{\frac{1}{2}}}{z}$, and $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$; which (multiplying both by $z^{\frac{1}{2}}$) comes to $L^{\frac{1}{2}}$, and $\sqrt{2} \times z^{\frac{1}{2}}$, or $\sqrt{L}$, and $\sqrt{2z}$; so that $V : W :: \sqrt{L} : \sqrt{2z}$. That is, the Velocity in a *Conick Section*, at the greatest or least Distance from the *Focus*, is to the Velocity in a Circle at the same Distance, in the subduplicate *Ratio* of the Parameter to that Distance doubled.

C O R.

C O R. IV

Let LD be the shorter Semi-axis of an Ellipse, then the Expression of the Velocity for an Ellipse at the Extremity of the shorter Axis, that is, at the mean Distance from the *Focus*, is $\frac{L^2}{D} = \frac{L^2}{\sqrt{2}}$. And the Expression for the Circle (being in general $\frac{\sqrt{2}}{z^{\frac{1}{2}}}$) comes (if z be put =

B) to $\frac{\sqrt{2}}{B^{\frac{1}{2}}}$. So that in this Case V is = W , since the Values of both are the same. That is, the Velocities of Bodies (revolving in Ellipses) at the mean Distances from the common *Focus*, are equal to the Velocities in Circles at the same Distances.

C O R. V.

Since V is always as $\frac{L^2}{y}$, therefore where L continues still the same, V is as $\frac{1}{y}$. That is, in the same, or equal Figures, or in unequal Figures whose Parameters are equal, the Velocity is Reciprocally as the Perpendicular from the *Focus* to the Tangent.

C O R. VI.

The Expression $\frac{L^2}{y}$ in the Parabola comes to $\frac{L^2}{y}$ because (by Cor. to Lemma II. foregoing) the Perpendicular from the Focus to the Tangent, viz. y is ever in the subduplicate Ratio of k , the Distance from the Focus to the Point of Contact. Or because in speaking of one and the same Figure, L is a standing Quantity, therefore V is ever as $\frac{L^2}{y}$. That is, in the Parabola

the Velocities are in the subduplicate Ratio of the Distances from the Focus Reciprocally. In the Hyperbola they will be in more than this Ratio, and in the Ellipse in less; the Reason of which is the different Proportion of the Perpendiculars to the Distances in those Curves.

C O R. VII.

The Expression for any Conick Section being $\frac{L^2}{y}$, and that for a Circle (whose Radius is $= z$) being $\frac{\sqrt{2}}{z^2}$; 'tis plain then (whatever Di-

stance z represents, whether equal to that in the

the *Conick Section*, or greater or less than it) that the Velocities are as $\frac{L}{y} : \frac{\sqrt{2}}{z}$, that is, as $L \times z : \sqrt{2} \times y$, or as $\sqrt{L \times z} : \sqrt{2} \times y$, that is, as $\sqrt{L \times z} : y$. Now $\sqrt{L \times z}$ is a mean Proportional between the Parameter of the *Conick Section*, and z the Radius or Distance in the Circle. So that this general Conclusion is to be inferr'd, that the Velocity in a *Conick Section* at any Distance from the *Focus*, is to the Velocity in a Circle at any other Distance (or whose Radius is = to any other Distance from the *Focus*) as a mean Proportional between that other Distance or Radius, and half the Parameter, is to a Perpendicular drawn from the *Focus* to the Tangent at the Point of Distance in the *Conick Section*. That is, z being the Distance in (or Radius of) a Circle, in which the Velocity is express'd by w , and V being the Velocity in a *Conick Section* at any Distance, equal or unequal to the former; the Conclusion in our Symbols stands thus, $V : w :: \sqrt{L \times z} : y$. Now, from this general Conclusion, all that relates to the Proportion of the Velocities in *Conick Sections* and Circles, may easily and naturally be inferr'd.

As,

As, 1. If z , or the *Radius* of the Circle, be equal to the Distance (from the *Focus*) in the *Conick Section*; then it follows, that the Velocity in a *Conick Section* is to the Velocity in a Circle at the *same Distance*, as a mean Proportional between that *same Distance* (or *Radius*) and $\frac{1}{2}$ the Parameter, to a Perpendicular from the *Focus*, to the Tangent at the Point of Distance in the *Conick Section*. This holds in all the *Conick Sections*. But the rest that follow, all except one, referring particularly to the Parabola, I note here, that in this Curve the Expression of the Distance from the *Focus* to any Point in the Curve is $= \frac{1}{4} L + x$ (x being the correspondent Absciss;) as also, that the Quantity y , the Perpendicular from the *Focus* to the Tangent at any Point, is $\sqrt{\frac{1}{16} L^2 + \frac{1}{4} Lx}$; both which are plain from the *Calculus* in the 2d *Lemma* to *Prob. XI*. Therefore to proceed,

2. In the Parabola, if we put $z = \frac{1}{4} L + x$; then $Lz = \frac{1}{4} L^2 + Lx$; but $yy = \frac{1}{16} L^2 + \frac{1}{4} Lx$, therefore in this Case $Lz = 4yy$, and $\frac{1}{2} Lz : yy ::$

$2 : 1$, and $\sqrt{\frac{1}{2} Lz} : y :: \sqrt{2} : 1$, or $V : w :: \sqrt{2} : 1$.

1. That is, the Velocity in a Parabola at any Distance from the *Focus*, is to the Velocity in a Circle at the *same Distance*, as $\sqrt{2} : 1$. Note, I say, at the *same Distance* in the Parabola for here $\frac{1}{4} L + x$ is put $=$ to z , the *Radius* of the Circle.

Q

3. If

3. If in the Parabola we put $z=y=\frac{1}{4}L$: Then $\sqrt{\frac{1}{2}Lz} : y :: \sqrt{2} : 1$, and consequently $V : w :: \sqrt{2} : 1$, in this Case also. That is, the Velocity in a Parabola at the *least Distance* from the *Focus* is to the Velocity in a Circle at the *same Distance*, as $\sqrt{2} : 1$. Note again here, I say, the *least Distance*; because 'tis put $=$ to $\frac{1}{4}$ of the Parameter, or because the Distance is supposed to be coincident with the Perpendicular from the *Focus* to the Tangent, which cannot be but when 'tis the least. Also z , the *Radius* of the Circle, is here supposed equal to the Distance in the *Section*.

4. In any *Conick Section*, if we put $z = \frac{1}{2}L$: Then $\sqrt{\frac{1}{2}Lz} = \frac{1}{2}L$; consequently $\sqrt{\frac{1}{2}Lz} : y :: \frac{1}{2}L : y$, or $V : w :: \frac{1}{2}L : y$. That is, the Velocity any where in a *Conick Section*, is to the Velocity in a Circle at the Distance of $\frac{1}{2}$ the Parameter, as $\frac{1}{2}$ the Parameter is to a Perpendicular from the *Focus* to the Tangent at the Point of Distance in the *Conick Section*. This is general for all the *Conick Sections*, there being nothing supposed in the *Calculus* that does restrain or limit the Property to any one Particular.

5. If in the Parabola we suppose $z = \frac{1}{8}L + \frac{1}{4}x$. Then $Lz = \frac{1}{8}L^2 + \frac{1}{4}Lx$, and consequently $\frac{1}{2}Lz = \frac{1}{16}L^2 + \frac{1}{8}Lx$, which is yy (as we hinted before.) Therefore $\sqrt{\frac{1}{2}Lz} = y$, and therefore $V = w$. That is, the Velocity in a Parabola is every where equal to the Velocity in a Circle at $\frac{1}{2}$ the Di-

Distance, or in a Circle whose *Radius* is $\frac{1}{2}$ the Distance from the *Focus*, in the Parabola. Note, I say, whose *Radius* is $=$ to $\frac{1}{2}$ the Distance in the Parabola; for the Distance being universally expressed by $\frac{1}{4}L + x$, I here put the *Radius* $z = \frac{1}{8}L + \frac{1}{2}x$.

And universally, if we we make $z : \frac{L}{4} + x :: n : 1$, and so $z = \frac{nL}{4} + nx$. Then will $\sqrt{Lz} =$

$\sqrt{\frac{nL^2}{8} + \frac{nLx}{4}}$; and consequently universally $V : v ::$

$\sqrt{\frac{nL^2}{8} + \frac{nLx}{4}} : y$. That is, the Velocity in the

Parabola will be to the Velocity in a Circle (whose *Radius* is n times the Distance in the

Parabola) as $\sqrt{\frac{nL^2}{8} + \frac{nLx}{4}} : y$. From whence particular Cases may be deduced at Pleasure.

And thus much for this admirable *Theorem* relating to the Velocities of Bodies, together with its *Corollaries*.

P R O B. XII.

Any Point being taken at Liberty in the Axe of an Ellipse, 'tis requir'd to find the Law of the Centripetal Force tending to the same. [See FIG. XIX. after the Preface.]

LET the Points $s. s.$ be taken of either Side the *Focus* of the Ellipse, which I make to be S . Then let the Line SP be drawn, and QT

let fall Perpendicular to it, and Qx parallel to the Tangent PR , intersecting SP in the Point x . Lastly, Let QR be parallel to SP , and the Intersection of SP with the Conjugate DC , be in the Point O . This being supposed with all the rest, as at *Prob. IX.* we have here the Triangles QTx and PPO , similar to one another, as there the Triangles QTx and FPE were similar. So the

$$QT^q \text{ is } = \frac{Qx^q \times PF^q}{PO^q}. \text{ But the last Ratio of } Qx \text{ to}$$

the Ordinate Qv , is a Ratio of Equality; consequently instead of Qx^q we may put Qv^q , and so QT^q

$$(\text{by substituting the Value of } Qv^q = \frac{Gv \times Pv \times CD^q}{PC^q},$$

$$\text{from Conicks), will } = \frac{PF^q \times Pv \times vG \times CD^q}{PO^q \times PC^q}. \text{ Again,}$$

the Triangles Pxv and POC are similar; there-

$$\text{fore } Px = \frac{Pv \times PO}{PC}; \text{ consequently } QR, \text{ which}$$

is $= Px$ from Parallels, is to be express'd by this Value. Therefore the Centripetal

Force, which is Reciprocally as $\frac{SP^q \times QT^q}{QR}$, is as

$$\frac{SP^q \times PF^q \times Pv \times vG \times CD^q \times PC}{Pv \times PO^3 \times PC^q}. \text{ And putting}$$

$BC^q \times CA^q$ (which is a standing Quantity) instead of $PF^q \times CD^q$, and instead of vG putting $2PC$;

by due Reduction the Expression comes to $\frac{SP^q}{PO^3}$,

which gives the Law of the Centripetal Force, viz. that 'tis Reciprocally, as the Square of the Distance from the Point s , divided by the Cube of

of the Line PO , intercepted between the Tangent PR and the Line DC , which is Conjugate to PC . From hence,

C O R. J.

If the Point s coincide with S , the *Focus* of the Ellipse; then the Line PO will also be coincident with PE , which is equal to AC , the Semi-transverse Axe. And therefore $\frac{sPq}{PO^3}$ will

in this Case come to $\frac{SPq}{CA^3}$; or because of CA^3 , a standing Quantity, it will be reduced to SP^3 . So that the Law of the Centripetal Force tending to the *Focus*, is this, that 'tis Reciprocally as the Square of the Distance from the *Focus*.

C O R. II.

If the Point s coincide with C , the Center of the Ellipse; then sP and PO also do both coincide with the Line PC , which is the Semi-Diameter for the Point P . Therefore the Ex-

pression $\frac{sPq}{PO^3}$ comes in this Case to $\frac{PCq}{PC^3}$, that

is, to $\frac{1}{PC}$. So that the Law of the Centripetal Force tending to the Center of the Ellipse, is this, that 'tis directly as the Distance from the Center. And both these Laws are the same with those found by particular Processes before.

I need not say, that this done for the Ellipse, may easily (*mutatis mutandis*, according to the Nature of the Curve) be applied to the Hyperbola; so that there's no Need of another Operation for that. But because the Application to the Parabola may not be quite so obvious, I think it proper to do that by it self.

P R O B. XIII.

Any Point being taken at Liberty in the Axe of a Parabola, 'tis requir'd to find the Law of the Centripetal Force tending to that Point. (See FIG. XX. after the Preface.)

LET the Point S be taken any where in the Axe AS; thro' which Point S let SO be drawn parallel to the Tangent PR, PO parallel to the Axe AS, and consequently a Diameter for the Point P. Qv an Ordinate to PO, and consequently parallel to PR, meeting also SP in x, and PO in v. Lastly, Let QR be parallel to SP, and the Lines QT and PF Perpendiculars from the Points Q and P, to the Lines SP and SO. This premised, the Triangles QTx and PSF are similar. Therefore $QT = \frac{Qx^q \times PF^q}{PS^q}$

But the last Ratio of Qx to Qv (that is, when the P and Q come together) is a Ratio of Equality; so that then $QT^q = \frac{Qv^q \times PF^q}{PS^q}$. And if

L be the Parameter belonging to the Vertex P,

P, then $L \times Pv = Qvq$, and so $QTq = \frac{L \times Pv \times PFq}{PSq}$

But QR is = Px (from Parallels) = $\frac{Pv \times PS}{PO}$

(from the similar Triangles Pxv and PSO.)

Therefore $\frac{QTq}{QR} = \frac{L \times PFq \times PO}{PS^3}$; and consequently

$\frac{SPq \times QTq}{QR}$ (which is Reciprocally as the Centri-

petal Force) is also as the Quantity $\frac{L \times PFq \times PO}{PS}$

And this is the *Law* of the Centripetal Force tending to the Point S taken any where in the Axe.

C O R.

Therefore if the Point S coincide with the *Focus* of the Parabola, PS will be = PO; for in that Case PS will = SG from the Nature of the Parabola, but SG is = PO from Parallels, therefore PS = PG. So that the Expression $\frac{L \times PFq \times PO}{PS}$

comes in this Case to $L \times PFq$. But SL being a Perpendicular from the *Focus* to the Tangent, PF is equal to it, from Parallels; and therefore PFq is ever as SP, from the Nature of a Parabola. So that $L \times PFq$ is still as $L \times SP$. But also from the Nature of the Parabola, L is = $4 SP$; therefore $L \times SP$ is ever as SPq . So that the Centripetal Force (tending to the *Focus*) is ever as the Square of the Distance from it, Reciprocally.

I am aware that that there may be more general Investigations of the Laws of the Centripetal Force than these, and I am acquainted with some of them; but what I have here said is sufficient for my Purpose.

FINIS.

E R R A T A.

The Author's long Absence from the Town (occasion'd by his Indisposition) has made it necessary to trouble the Reader with something more of this Correcting Work, than otherwise should have been thrown upon his Hands. 'Tis desired, that he would candidly pass over the following Errata, or any other that occur, which have escap'd Notice.

PAge 11. line 26. read $m+n$. p. 17. l. 3. r. $sxx+dd$.
 p. 20. l. ult. r. Angle ADF. N. B. p. 28, & 32. The Figure should be as 'tis at p. 30, & 32. the Points C and K coinciding. p. 34. l. 4. r. Point H. *Ibid.* l. 7. r. Plane CH. *Ibid.* l. 8. r. Line AH. p. 36. l. ult. r. All and AF. p. 62. l. 12. r. axL . p. 74. l. 13. r. KB. p. 88. l. 1, 2. r. after the Stroke, the Sum of the Moments is $=Bx+bz$. Therefore, &c. p. 95. l. 5. r. Therefore. p. 112. l. 13. r. AC. p. 115. l. 5. r. AG:DC. p. 126. l. 4. Proportional. p. 128. l. 28. r. Perimeter. N. B. p. 138. The latter Period is falsely put on the Lemma it self, whereas it begins what follows after it. *Ibid.* l. 20. r. $d^2=2rx$, and $D^2:d^2::2RX:2rx$, p. 139. l. 2. r. $2RX=A^2$, and $2rx=a^2$; so that $A^2:a^2::2RX:2rx$; and when the Points G, g coincide, or $2R=2r$, then 'tis $A^2:a^2::x:x$. So that the, &c. p. 159. l. 19. r. yx . p. 172. l. 20.

$\frac{QT}{QR}$ as SP; and because, however the Angle of Contact is changed, the Subtense QR is changed also in the subduplicate Ratio of RP or QT, therefore $\frac{QT}{QR}$ shall ever be in the same Ratio of SP, and therefore $\frac{SP \times QT}{QR}$, &c. p. 202. l. 19. r. $ES=EL$. p. 205. l. 1. r. Prob. X. *Ibid.* l. 20. r. $ES=EL$. p. 182. l. 9. r. and PS. *Ibid.* l. 20. r. $\frac{PS \times Qr}{QR}$.

N. B. Ever after in that Problem read PS instead of PC. p. 203. l. 15. r. $GuxPv$. p. 173. Supply the Letter G in FIG. XI. at the End of PC drawn out to the Ellipse.



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